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## FORWARD

Although the specific purpose of the present study was to investigate the development of mental structures for classification, ordering, and number conservation in a particular group of Colombian children, a much broader and deeper aim brought me to this particular line of research; an aim which transcends even the more practical goals of enabling students to think more logically, to understand scientific classification systems, and to be more successful in areas of mathematics such as number concepts, place value, and the use of the base-10 system. During my sophomore year of college I chanced upon the Writings of Bahá'u'lláh, the Prophet-Founder of the Bahá'í Faith. I became particularly intrigued and eventually enthralled by His explanation of God's purpose for the life of humankind on this earthly plane:

Having created the world and all that liveth and moveth therein, He, through the direct operation of His unconstrained and sovereign Will, chose to confer upon man the unique distinction and capacity to know Him and to love Him--a capacity that must needs be regarded as the generating impulse and the primary purpose underlying the whole of creation. (Bahá'u'lláh, 1950, p. 65)

Bahá'u'lláh further explains that there are two principal means or two "books" through which humanity can come to know the Creator--the Book of Revelation, i.e., the scriptures of the revealed religions, and the "book of creation," i.e., through nature, which includes human beings. Regarding the "book of creation," He states:

Upon the inmost reality of each and every created thing He hath shed the light of one of His names, and made it a recipient of the glory of one of His attributes. Upon the reality of man, however, He hath focused the radiance of all of His names and

attributes, and made it a mirror of his own Self. Alone of all created things man hath been singled out for so great a favor, so enduring a bounty. (Bahá'u'lláh, 1950, p. 65)

During this same period of my life, through my study of the Anisa Model, a comprehensive, developmental system of education, I was introduced to the ideas of the British philosopher, mathematician, logician, and educator Alfred North Whitehead. In his philosophy of organism Whitehead (1978) describes creation as being composed of hierarchically structured "societies" or ontological levels:

The notion of a society which includes subordinate societies and nexus with a definite pattern of structural inter-relations must be introduced. Such societies will be termed 'structured.'

A structured society as a whole provides a favourable environment for the subordinate societies which it harbours within itself. Also the whole society must be set in a wider environment permissive of its continuance. Some of the component groups of occasions in a structured society can be termed 'subordinate societies.' But other such groups must be given the wider designation of 'subordinate nexus.' (p. 99)

He further explains that

the physical world exhibits a bewildering complexity of such societies, favouring each other, competing with each other.

The most general examples of such societies are the regular trains of waves, individual electrons, protons, individual molecules, societies of molecules such as inorganic bodies, living cells, and societies of cells such as vegetable and animal bodies. (p. 98)

In "A Summary Statement on the Anisa Model" (Jordan, 1974) the foundational philosophical principles include this Whiteheadian concept of interlocking, hierarchically organized ontological levels:

The philosophy underlying the Model . . . accepts the principle of hierarchical structuring as primary expression of order and beauty in the universe [and] defines the basic order of the universe in terms of different hierarchically arranged ontological levels and places man at the apex of all living creatures. (p. 24)

In the course of my studies I became intrigued by this principle, especially in relation to the statements of Bahá'u'lláh regarding the purpose of human life and creation itself. I reasoned as follows: "If one of the main purposes of earthly life is to grow in our knowledge of the Creator via the increase of our knowledge of His attributes as manifested in His creation, and, if one of the principle characteristics of His creation is that it is hierarchically organized, then it would be a marvelous gift of education to enable students to more readily and clearly perceive the logical, hierarchical organization within all things." This thought led me to the investigation of how the ability of hierarchical classification develops in children and how it contributes to their understanding of various symbol systems and subject content. Although this present study does not investigate hierarchical classification directly, it does focus on its developmental prerequisites: preclassificatory collections and class inclusion.

The idea that pedagogical measures can be taken to facilitate and refine a child's understanding of the structure of his or her world, I find to be extremely exciting. And to produce a graduate who can enter any field, any organization, or any society and be able to extract its underlying, hierarchical organization and through this to more deeply realize and appreciate the beauty and order which undergird God's creation, I consider to be a supremely worthwhile educational goal.

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## I. INTRODUCTION

The present study focused on the development of logical, mental structures in a sample of Colombian children between the ages of 5 and 8 using the theoretical framework set forth by the Swiss scholar, Jean Piaget and the Piagetian assessment tasks developed by University of Iowa researcher and teacher trainer, Darrell G. Phillips. (See Appendix H for a biographical sketch of Jean Piaget.)

Four logical structures were chosen, one pre-operational and three concrete operational. The pre-operational structure selected was "collections" which is pre-classificatory in nature. The concrete operational structures consisted of two logical groupings: "primary addition of classes" (also called "class inclusion") and "addition of asymmetrical relations" (also known as "ordering" or "seriation by length"); and one number group, "additive group of whole numbers," which was partially assessed in the present study via the Conservation of Number Task<sup>1</sup>. The latter three structures appear as LG<sub>1</sub>, LG<sub>5</sub>, and N<sub>1</sub> respectively in Appendix A: Chart of Concrete Operational Structures (D. G. Phillips, 1996, p. 53). The nature of each of these structures will be discussed later in this thesis.

The impetus for the present study arose from two concerns: (1) this author's long-standing frustration with third grade students' difficulties with math concepts such as rounding, estimating, base-10, and place value and with concepts involving classification such as the relations amongst urban / rural, city / neighborhood, and the scientific classification of animals<sup>2</sup>; and (2) the need to apply cross-culturally the

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<sup>1</sup> For the purpose of clarity, the particular assessment tasks used in this study will be capitalized.

<sup>2</sup> Stimulated by an article by Dr. Donald T. Streets (1976) regarding the importance of



standardized assessment tasks developed by Darrell G. Phillips and his colleagues.

Data for the present study were collected using the Piagetian clinical interview technique in which subjects are individually presented with a task which requires them to act upon a set of objects in order to show their response and, in most tasks, the subjects are asked to verbally justify their response.

The principal research questions focused on differences of task performance amongst subjects of the same age range, amongst different age ranges, and between males and females.

This introductory section will set forth the justification for the present study, review the basic ideas of Piaget, describe the various levels of cognitive development and their hierarchical organization with special attention given to the cognitive structures investigated in the present study, discuss the problems inherent in Piaget's stage theory of development, place Piaget in a broader philosophical and theoretical context, and state the research problem and questions of the present study.

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classification, seriation, transitivity, and conservation for number concept formation, during the course of a three-year period, I tried to use direct teaching combined with manipulative and experiential activities to develop in my students these abilities and the targeted concepts. Many excellent activities were invented which incorporated the use of Venn hoops, hierarchical nesting cups and hoops, student-constructed number lines, base-10 block constructions as high as one million, games for rounding and classifying, field trips, and many others. However, it was not until I was trained in the Developmental Activities Program created by Darrell G. Phillips and Dale R. Phillips that I began to understand what could have been missing: the students might have needed more time to develop the logical-mathematical, mental structures which would have enabled them to understand these concepts. Some students must have had the structures while others evidently did not. Moreover, I was unsuccessful in directly "teaching" those who were having difficulty understanding. Now I see that I needed a more individualized program which allowed for developmental differences and which provided activities for developing the needed structures.

### Justification

The primary justification for the present study consisted in the information it provided regarding the development of four mental structures in a sample of Colombian children. These mental structures are considered to constitute the basis for understanding the concepts of number, especially the cardinal and ordinal aspects of number, place value in general, and the base-10 system in particular. Regarding the importance of class inclusion and seriation in relation to number, Piaget (1965) stated:

Our hypothesis is that the construction of number goes hand-in-hand with the development of logic, and that a pre-numerical period corresponds to a pre-logical level. Our results do, in fact, show that number is organized, stage after stage, in close connection with the gradual elaboration of systems of inclusions (hierarchy of logical classes) and systems of asymmetrical relations (qualitative seriations), the sequence of numbers resulting from an operational synthesis of classification and seriation. In our view, logical and arithmetical operations therefore constitute a single system that is psychologically natural, the second resulting from the generalization and fusion of the first. (p. viii)

Concerning the importance of "collections" as the foundation of classification, D. R. Phillips (1991) emphasized the point that "an understanding of base 10, place value, and number depends on having classification structures in place. These, in turn, are built on two pre-classification levels called graphic and non-graphic collections" (p. 108).

The fourth mental structure investigated in the present study was conservation of number which, according to D. G. Phillips (1996) is

the basis of our entire mathematical system. When you find a child who does not conserve number, it is not surprising that numbers in a book do not make sense to him/ her. After all, the number of things can change depending upon their arrangement on the page. These are the children who, bereft of adequate concrete experiences in the early grades, grow up to be the adults who still count on their fingers. (p. 437)

Elsewhere he explained that "the child who does not conserve number will not believe that one ten is the same as 10 ones when they are spread out" (p. 302).

Another justification for the present study was the need to extend the data-base of research which utilizes the standardized protocols developed by D. G. Phillips and his colleagues (1996). One problem of doing research based on Piaget's work has been that his publications include no task protocols and only pieces of descriptions of interviews. This has led to many discrepancies in the reported investigations. D. G. Phillips, who has spent the past thirty years organizing the research of Piaget and carrying out research studies of his own, has addressed this problem by creating uniformity of protocols, objects, and scoring criteria. His research has been extensive. He has conducted or directed 43 research studies involving more than 4100 subjects including a three-year longitudinal study (Phillips, 1989). (See Phillips, 1996, pp. 472-473, for a complete bibliography of these studies.) The results have confirmed several Piagetian notions such as: the thought processes of children are qualitatively different from those of adults; mental structures are developed by children acting on objects; the development of cognitive processes can be facilitated by a trained adult who asks the child thought-provoking questions while he/ she is interacting with the objects (especially during transition phases); the structures cannot be taught through demonstration or explanation (i.e., by social transmission alone); certain objects and activities facilitate the development of particular mental structures better than others;

children develop mental structures at different rates and at different times; not all people (even adults) have developed all cognitive structures (because the structures are not developed simply by biological maturation); and, many of the structures are hierarchically organized and are developed in sequence, while other structures are developed simultaneously with one another. The studies directed by Dr. D. G. Phillips (1996, pp. 472-473) were carried out mainly in Iowa and other Midwestern states. One of his studies took place in South Africa (Alport, 1982). Studies such as the present one, carried out in other cultures using his unified system of protocols, objects, and scoring criteria, will contribute to the construction of an international data-base which will enable educators around the world to compare their assessments with one another and to place their endeavors on a scientific foundation.

A closely related justification concerns the implication of Piaget's work that the development of mental structures follows the same pattern and sequence in all human beings everywhere, regardless of race, culture, or social class, while allowing that the structures are developed at different rates and to different extents in different cultures. David Elkind (1988) stated:

In recent years evidence has been accumulating in support of the stages described by Piaget. The tests he devised have now been used in more than half the countries of the world with amazingly comparable results. Children all over the world go through these stages at least up to adolescence. (p. 97)

In this author's research of the extant literature in English and the library holdings of Universidad del Norte in Barranquilla, the most reputable university on the North Coast of Colombia, no published studies were found regarding Piagetian-based research

carried out in Colombia.<sup>3</sup> Hence, the present study involving Colombian subjects will be a valuable addition to a growing body of coherent research and could be used to confirm or disconfirm the hypothesis that there are developmental universals in the sequence of the formation of mental structures while recognizing diversities of rate and intensity amongst various cultures and social strata.

### Jean Piaget's Theory of Cognitive Development

In all psychology, few theorists are as important as Jean Piaget, who forged the single most comprehensive and compelling theory of intellectual development.

--William C. Crain (1985, p. 88)

#### Overview of Basic Ideas

#### Factors Influencing Development

Piaget identified four interacting factors which influence development: maturation, social transmission, experience, and equilibration. Maturation concerns those aspects of development such as biological growth, perceptual competence, and psychomotor competence which, if a "normal" environment is provided, will unfold naturally (Jordan, 1981b). Social transmission concerns those objects of knowledge which can be learned from others via imitation, language, cooperative learning and direct teaching. Language itself is acquired through social transmission along with all of the language-encoded information which a culture deems worthy of passing on to future generations. Experience, for Piaget, is of two types: physical

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<sup>3</sup> The education department at Universidad del Norte has carried out an extensive, ten-year study of the development of formal operations amongst the youth on the North Coast of Colombia, but the results are still in the process of being published.

experience--interaction with the physical environment, and logico-mathematical experience—reflection on abstractions drawn from actions upon the physical objects. Equilibration is "the process of bringing assimilation and accommodation into balanced coordination" (Flavell, 1963, p. 239). Assimilation means taking "information from the environment into our existing mental structures" (D. R. Phillips, 1991, p. 21). Accommodation refers to the change brought about in the mental structure so that it will "fit with the incoming information" (D. R. Phillips, 1991, p. 21). Later in this section these factors will be discussed further.

### Piaget's Focus of Inquiry

The child explains the man as well as and often better than the man explains the child (Piaget and Inhelder, 1969, p. ix).

In their explanation of genetic psychology as "the study of the developmental processes that underlie the mental functions" Piaget and Inhelder (1969) delineated the origin of their principal research question:

The study of logical thinking, its operation and structures, in the completed state found in the adult led some authors (German *Denkpsychologie*) to believe that thinking was a "mirror of logic." Psychologists eventually began to wonder whether logic was innate or resulted from a gradual development. To solve problems of this kind they turn their study to the child and in so doing promote "child psychology": "genetic psychology" becomes an essential tool of explicative analysis to solve the problems of general psychology (p. viii-ix).

According to Herbert P. Ginsburg and Sylvia Oppen (1988), "Piaget's primary

goal, then, could be defined as the study of children's gradual attainment of intellectual structures which allow for increasingly effective interactions with the environment" (p.13). John H. Flavell (1963) viewed Piaget's work as "the interpretation of developmental events within an epistemological, theory-of-knowledge framework" (p.262). He further stated that Piaget "has been primarily interested in the acquisition of concepts like classes, relations, number, space, time, and so forth: these 'grand and fundamental categories of experience'" (p. 251).

### Knowledge and Intelligence

For Piaget knowledge is not merely a copy of something which exists outside of the knower; something which can be passively absorbed by "osmosis" through the perceptual sense organs. Rather, knowledge must be constructed by the knower via his/ her actions on the object of knowledge:

"The subject only knows reality through his actions (and not merely through his perceptions)" (Piaget, 1972, p. 82).

But, to acquire knowledge, action alone is insufficient. The actions need to be guided by intelligence which Piaget viewed as being composed of intellectual structures such as classification; seriation; number; various conservations; and space, time, and measurement structures amongst others. Intelligence, i.e., "the mental structures possessed by an individual" (D. G. Phillips, 1996, p. 6), for Piaget, is not a given. The formation of structures is not guaranteed via the processes of maturation alone and structures cannot be acquired through social transmission. Intellectual structures, like knowledge, must be constructed by the knower. According to Piaget, "intelligence organizes the world while organizing itself" (as cited in Flavell, 1963, p. 62); hence, there are two things that are being organized or constructed: knowledge of the world

and intellectual structures.<sup>4</sup> Piaget (1972) stated:

It is clear that knowledge, with its logico-mathematical and physical bipolarities, is formed on the plane of action itself as actions become coordinated, and subject and objects begin to differentiate themselves through the progressive refinement of mediating structures [emphasis added]. (p. 24)

### Structures

Piaget (1971) offered the following explanation of structures:

As a first approximation, we may say that a structure is a system of transformations. Inasmuch as it is a system and not a mere collection of elements and their properties, these transformations involve laws: the structure is preserved or enriched by the interplay of its transformation laws, which never yield results external to the system nor employ elements that are external to it. In short, the notion of structure is comprised of three key ideas: the idea of wholeness, the idea of transformation, and the idea of self-regulation. (p. 5)

D. G. Phillips (1996) described structures as "mental processes that humans use to act on, manipulate, and make sense of data" (p. 6). He further distinguished these processes from information: "Structures are not content facts; structures *act on* content

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<sup>4</sup> It is not uncommon for expounders of constructivism to lose sight of this dual construction. What is usually omitted is the construction of structures, or "constructs" in the terminology of George Forman (1993) who stated, "Unfortunately, educators have treated constructivism as a process without a clear conception of these structural criteria." For example, one author wrote, "This 'constructivist' viewpoint rests on the assumption that children mentally 'construct' knowledge through reflection on their experiences. . . . The teacher also . . . collaborates with children in constructing knowledge" (Monighan, 1993, pp. 19-20).



facts" (p. 6). They are composed of operations. An operation is "an action on objects, which is then internalized, becomes reversible and is coordinated with other operations, that is, incorporated into a structure" (1996, p. 11).

Jordan (1981a) used an analogy of a template to illumine the nature of structures (although he preferred the term "process"):

Suppose we call these mental operations, templates, which may in many cases be applied to various aspects of the environment or to the body itself. . . . Templates . . . are formed through learning by repeated interactions with particular environments that afford opportunities for the appropriate differentiations, integrations, and generalizations that comprise a template (process). Once a template is formed, it can be activated or applied in a number of different ways. Each new way that it is applied represents a new generalization of it. . . . For example, classification is a template. (pp. 5-6)

Structures are formed via the interaction between the subject (including his/ her genetic endowment) and the environment. Piaget (1972) saw this same dynamic underlying both biological and mental development:

Present-day thinking on the phenotype shows this to be the product of an indissociable interaction, from embryogenesis onwards, between hereditary factors and the environment, so that it is impossible to trace a fixed boundary between the innate and the acquired, since between the two is found the region of the self-regulations characteristic of development; and this holds with even greater force on the level of cognitive behaviour. (p. 56)

Interaction with the environment, for Piaget, meant experience which can be

equated with mental or physical actions carried out on the world of objects. Piaget identified two types of experience or action: physical and logico-mathematical:

There are, in fact, two kinds of experience which are important from a pedagogical point of view. . . . First of all, there is what I call physical experience, and secondly, what I shall call logico-mathematical experience (as cited in Waite, p. 251).

Structure development begins with physical experience, but then the data or attributes of the objects are abstracted from the actions on the objects. Later, the subject, depending on his/ her stage of development, can manipulate or transform the objects in his/ her mind without the physical presence of the objects. Thus, experience or interaction with the environment, Piaget (1969) asserted, is essential for the development of mental structures:

Logico-mathematical concepts presuppose a set of operations [structures] that are abstracted not from the objects perceived but from the actions performed on these objects, which is by no means the same. (p. 49)

Elsewhere, Piaget clarified the type of experience that is needed:

Experience is always necessary for intellectual development . . . but I fear that we may fall into the illusion that being submitted to an experience (a demonstration) is sufficient for a subject to disengage the structure involved. But more than this is required. The subject must be active, must transform things, and find the structure of his own actions on the objects (as cited in Wadsworth, 1984, p. 195).

### Functional Invariants: Organization and Adaptation

In both biological and mental development and crossing all stages of development, Piaget identified two interdependent processes: organization and adaptation. It is through these processes that mental structures are built:

From the biological point of view organization is inseparable from adaptation: They are two complementary processes of a single mechanism, the first being the internal aspect of the cycle of which adaptation constitutes the external aspect . . . The "accord of thought with things" and the "accord of thought with itself" express this dual functional invariant of adaptation and organization. These two aspects of thought are indissociable: It is by adapting to things that thought organizes itself and it is by organizing itself that it structures things. (as cited in Flavell, pp. 47-48)

Organization then is the natural tendency for physical and mental systems to integrate lower-order processes into higher-order processes. On the biological level, for example, cellular processes are organized into tissue and organ-based functions. At the sensorimotor level, the grasping reflex is integrated with curiosity to produce groping, grabbing, exploration, and eventually simple experimentation with the physical objects (Flavell, 1963; Ginsburg & Opper, 1988; Wadsworth, 1984).

Adaptation is the predisposition for biological organisms and mental structures to adjust to the environment. It consists of two complementary sub-processes: assimilation and accommodation. According to Piaget (1972), "the idea of assimilation implies that of the integration of the given within a prior structure or even the formation of a new structure" (p. 22). Wadsworth (1984) defines it as "the cognitive *process* by which a person integrates new perceptual, motor, or conceptual matter into existing schemata or patterns of behavior" (p. 14). Piaget and Inhelder (1969) defined

accommodation as "the modification of internal schemes to fit reality" (p. 6).

When data from experience are assimilated into an existing structure after minor or major accommodations, equilibration is attained. For Piaget and Inhelder (1969) "intelligence constitutes an equilibration between assimilation and accommodation" (p. 58). When data from experience cannot coherently be assimilated into a structure disequilibrium is created. This contradictory data can be ignored for a time, or, it can eventually lead to the accommodation of a structure, even the synthesis of a new structure. For example, children often believe that the mass of a ball of clay is increased when it is rolled into a sausage shape and then decreased when further rolled into a string shape. With experience and reflection the child eventually comes to understand that the amount remains the same in spite of perceptual changes in width or length (Flavell, 1963; Ginsburg & Opper, 1988; Wadsworth, 1984).

Disequilibrium or "cognitive conflict," then, is a key cause for mental development. It leads eventually to higher-order levels of equilibrium. For D. G. Phillips (1996) equilibration is: the "factor contributing to the development of intelligence" which is "by far the most important"; "an internal type of mental organization"; "the mechanism by which the child coordinates; or balances, the factors of experience, maturation, and social transmission"; "an *action* carried out by the child alone"; the means through which "mental structures are modified and created" (p. 7).

In order to synthesize, summarize, and better understand these basic ideas, perhaps a metaphor would be helpful. Imagine a structure as a set of adjustable shelves used for organizing canned goods (content facts). As cans are received shelves are assembled (organized) so that the cans are ordered by height on an appropriately-spaced shelf (assimilated to the structure). At times, a particular shelf may need to be lengthened (a minor accommodation of the structure). At another time, when some unfamiliar, medium-sized cans are received, in order to organize the cans from largest at the bottom to smallest at the top, a major reorganization (adaptation) of

the shelves is necessary--perhaps even the insertion of new shelves (a major accommodation of the structure). After each adjustment or change, the balance and functionality of the shelving scheme is reestablished (equilibrium). When a particular-sized can is needed, retrieval (memory) is easy. If the cans (facts) had been received without any shelves they would be piled up at random (short-term, rote memory). Retrieval (recall) would be difficult. After the shelves have been assembled, they can be used for organizing other items also, such as boxes, cartons, or dishes (generalization/ transfer/ application of the structure to other situations, events, places, things, etc.) with only minor adjustments (accommodations). The shelves and the shelving process exemplify the three qualities of a structure: wholeness (a unified, purposeful storage system), transformation (randomly received cans are changed into an orderly array), and self-regulation (as unexpected-sized cans are encountered appropriate shelves and shelf-space are created).<sup>5</sup>

### Levels of Cognitive Development

Piaget viewed development as being a slow, continual process, which, for conceptual purposes, he divided into four stages or periods which can also be considered as hierarchically nested levels or types of thinking each of which manifests certain distinctive features: sensory-motor thought, pre-operational thought, concrete-operational thought, and, formal-operation thought. The order of the levels is

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<sup>5</sup> Although educational implications will be discussed later, I do want to point out how this metaphor clarifies the curriculum issue of content (culture-based information) versus process (structure formation); namely, that taking time to build the shelves (structures) pays off later in the easier retrieval (memory) of cans (facts). Hence, a strong process program also strengthens the content program because mental structures help the learner to understand, organize, encode, and retrieve information much more efficiently than rote memory.

invariant but the age of onset varies (D. G. Phillips, 1996, pp. 8-10; D. R. Phillips, 1991, pp. 12-16). In the following discussion emphasis will be placed on the types of thought and the cognitive structures involved in the present study.

### Sensori-Motor Thought

Sensori-motor thought (birth to two years) is characterized by these developments:

- patterning of reflexive movements such as the sucking and grasping schemes;
- coordinating reflexive movements and perceptual functions for practical use, e.g., seeing objects, then seeing and grabbing objects, then seeing, grabbing and sucking objects;
- structuring of object permanence, i.e., knowing that an object still exists even though it is outside of the perceptual field;
- structuring of space, e.g., differentiating self from a three-dimensional world containing separate, three-dimensional objects;
- structuring of physical causality, e.g., understanding that to move an object at a distance there must be some physical connection with it;
- growth of means-end relations marking the beginnings of goal-oriented behavior, e.g., pulling on a blanket to obtain an out-of-reach object which is on top of the blanket;
- inception of intentional behavior marking the beginning of consciousness and human intelligence, e.g., looking at a distant object and then pulling on a cord to move it, or, arching the back and then falling back as a means of rocking the cradle;
- experimental, trial and error behavior initiating empirical intelligence, e.g., repeatedly dropping various objects to discover results (D. G. Phillips, 1996; Piaget, 1972; Piaget & Inhelder, 1969; Waite, 1975).

### Pre-operational Thought

Pre-operational thought (emerging at 18 to 24 months) features these characteristics:

- symbolic representation of objects, i.e., symbols, acts, or objects representing other objects;
- representational thought;
- perceptually-bound thought;
- ability to consider only static arrangements;
- inability to consider transformations or changes between static states, i.e., the world is dealt with as a "slide show" rather than as a "moving picture";
- inability to go back and forth between successive states, i.e., the lack of reversibility (D. G. Phillips, 1996; Piaget, 1972; Piaget & Inhelder, 1969; Waite, 1975).

### Graphic and Non-Graphic Collections:

The present study examined one pre-operational function called collections which prepares the child for building operational classificatory structures. It is composed of two levels: figural collections and non-figural collections (Piaget, 1972, p. 32). D. G.

Phillips (1996) referred to these levels as graphic collections and non-graphic collections. A graphic collection

is concerned with partitive membership, that is, a particular object is placed with another object because it belongs to, or fits, a particular shape, or spatial configuration. The child at this level is concerned primarily with the overall appearance of the arrangement of the objects [such as attribute shapes]. Typically these arrangements are such that each piece is completely visible,

nothing is covered, and there are no piles of objects. (p.31) [For sample arrangements, see Appendix B.]

According to D. G. Phillips (1996), the primary characteristic of the non-graphic-collections level

is that of class membership. This class membership is not the same as class inclusion since membership is a relation between a particular element (x), or object, and a collection, or class (A), while a relationship of class inclusion involves the relation between one class (A) and another class (B), and is concerned with relationships such as  $A > B$ . . . . Non-graphic collections are still subordinate to the principle of spatial proximity among elements even though they are free from the condition of a definite shape. Non-graphic Collections are not true classes. (p. 31) [See Appendix B for sample arrangements.]

At the pre-classificatory, "collections" level children begin to make collections of objects that go together due to function, such as "cooking things"; due to spatial proximity, such as: "This square and this triangle could be pushed closer together to make a house."; due to shape, such as: "I think I'll line up, or stack up all of the small rectangles, or, just these that happen to be close together." Later they will be able to group things that are not close together in time or space, such as: "My favorite things to do" or "My favorite places to go." The ability to organize collections contributes to and is a necessary foundation for the eventual development of class inclusion, and for this reason it was chosen as one of the four types of logical thinking to be investigated in the present study (D. G. Phillips, 1996; Piaget, 1972; Piaget & Inhelder, 1969).



## Concrete-Operational Thought

Concrete-operational thought (starting tentatively at 6-8 years) is characterized by four abilities: (1) the use of logic to correct perceptual miscues of reality; (2) conservation of quantities (maintaining one attribute of a physical entity constant in spite of transformations of other attributes; (3) reversibility of thinking (returning mentally to a specific starting point); and (4) the ability to decenter; that is, to coordinate more than one dimension, perspective, or viewpoint simultaneously. Concrete-operational thought is called "concrete" because the person's thought is focused either on physical objects in his/ her presence or on the "internalized" actions he/ she had carried out on physical objects at a previous time. Hence, concrete operational thought can be abstract, but the content of the abstract thought is still objects and their transformations (D. G. Phillips, 1996).

Categories of concrete-operational structures include classes, relations, number, space (topological, projective, and Euclidean), measurement, and time. Unlike the formal-operations level, the structures formed at this level function in isolation from one another; that is, they are not yet coordinated into an over-arching system. Appendix A contains a chart created by Darrell G. Phillips to illustrate some of the concrete-operational structures. Groupings are mental structures which are characterized by four logical operations: associativity, reversibility, composition, and identity; and by semi-lattice properties<sup>6</sup> (tautology and either resorption or absorption). All Groupings deal with classes and relations (D. G. Phillips, 1996). Logical groupings and infralogical groupings, according to Inhelder and Piaget (1969), operate on (or

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<sup>6</sup> A lattice is defined by Flavell as "a structure consisting of a set of elements and a relation such that any two elements have one g.l.b. [greatest lower bound] and one l.u.b. [least upper bound]" (Flavell, 1963, pp. 172-173 [See this reference for further explanation of terms and concepts.]).

transform) different categories of "objects":

The difference between what is logical (or pre-logical) and what is sub-logical [infralogical] is simply that the former deals with the relations between discontinuous elements [such as pattern blocks] while the latter relates to elements forming part of a spatial continuum [such as a relief map]. (p. 282)

The present study dealt with one pre-logical function, collections, (hence, it is not on the chart of logical groupings) and two logical grouping (LG) structures--class inclusion (primary addition of classes [LG<sub>1</sub> on the Appendix A chart]) and ordering by length (addition of asymmetrical relations [LG<sub>5</sub> on the Appendix A chart]). No infralogical structures were investigated.

Also, one aspect of the number groups (N), conservation of number (additive group of whole numbers [N<sub>1</sub> on the Appendix A chart]), was studied. The difference between groupings and groups is an important one (above and below the dotted line on the chart, respectively). Although they both share the logical properties of composition, reversibility, identity, and associativity, in the group structures elements are counted as individuals and in the grouping structures the content is dealt with only as classes and relations. For example, in a grouping structure it might be inferred that there are more students in an elementary school than there are in the third grade without knowing exactly how many more. In a group structure, however, what is important, for example, is knowing that putting three elements together with five elements gives the exact same quantity as putting together four elements and four elements. In other words, groupings are qualitative in nature and groups are quantitative.

### Class Inclusion:

The class inclusion structure deals with the relation between classes and subclasses. Inhelder and Piaget (1969) explained that "classes may be defined both by their 'intension' and by their 'extension'" (p. 7). The intension of a class is the set of attributes that the members of a class have in common. The class of red, triangular, plastic blocks share the features of "redness" of color, triangularity of shape, hard "plastic-ness" of substance, and "blockishness" of type. The extension of this class signifies all the members of this class, that is, the "field of application" of the properties defined by the intension. In this case it would be all red, triangular, plastic blocks. When, for example, blue, triangular, plastic blocks are added to the set, the extension of the red triangles no longer extends to "all" of the blocks, but, rather, to only "some" of the blocks. Hence, the terms "all," "some," "one," and "none" designate the quantity of the extension of a subordinate class in relation to a superordinate class. Inhelder and Piaget (1969) explained:

A system of logical classes involves a set of similarity and difference relations which, together, yield an intensional definition of every class and sub-class: (predicates such as "green" or "solid" are never absolute and invariably point to relations of similarity: "co-green" or "co-solid"). On the other hand, the elements or individuals qualified by these relations are quantified by means of extensive quantifiers "all," "some" (including "one") and "none." It is important to note that the intension of each class uniquely determines its extension. Thus intension and extension are always in correspondence, so that whenever one is known the other can be determined. (pp. 44-45)

For example, in the set of red and blue triangles described above, if we define the intensional qualities of the sub-class as "triangles," the extension of the sub-class will

be "all", i.e., the sub-class is identical to the class. If the intensional attributes are "red triangles," the extensive quantifier would be "some." And if we defined the subclass as "yellow triangles," the extensive quantifier would be "none."

If children cannot correctly apply extensive quantifiers such as "all" or "some" to the relation between a sub-class and a class, then they will have difficulty with class inclusion tasks. For example, when presented with four daisies and nine roses, they will be able to indicate which belong to which subordinate class--daisies and roses. They will also be able to indicate that both the daisies and the roses belong to the superordinate class of flowers. But when they are asked, "Are there more roses or more flowers?" and requested to explain their answer, they will respond with something like, "There are more roses because, look, there are only . . . (counts silently) four flowers." They cannot deal with superordinate and subordinate classes simultaneously (an example of centration on one dimension or classification level). They can only compare daisies and roses, not roses and flowers. Perception gets in the way of logic. The "class-inclusion" children, on the other hand, will respond something like, "There are more flowers because roses are flowers and so are daisies. Roses are only part [some] of the flowers. When you put them all together, there are more flowers than roses or daisies by themselves" (based on example of D. R. Phillips, 1991, p. 110).

In relation to number, according to D. R. Phillips (1991), research indicated that before beginning to work on place value a child should have developed the class inclusion structure to the point at which his/ her reasoning indicates complete generalization of the structure. (This presumes that graphic and non-graphic collections have been developed.) He/ she should also have developed two other structures: seriation with correspondence and conservation of number.<sup>7</sup> Phillips

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<sup>7</sup> In addition to these structures, which are the focus of the present study, the child, according to D. R. Phillips (1991, p. 301), should: have worked through [number]

further stated that "class inclusion allows the child to realize that all numbers below a certain number are contained within it" (p. 302). Hence, a typical error:  $43 - 17 = 36$  would be less likely because a "class includer" would more easily note that 17 and 36 cannot both fit within 43 (p. 301).

Regarding the importance of this structure, D. R. Phillips stated:

Class inclusion, or primary addition of classes, opens up many relationships that were previously unavailable to the child. Among them are the relationships among numbers, e.g., "7 contains 4 and 3, and you can't do  $4 - 7$  because there is no 7 in 4!" The child is able to see the parts and the whole at the same time. All additive number relations depend upon inclusion as part of their structure (e.g., the child cannot deal with, "'one ten' is the same as 'ten ones'" before he/ she has class inclusion), (p. 111)

Ordering by Length:

Ordering by length, or seriation, as defined by Inhelder and Piaget (1969), is "the product of a set of asymmetrical transitive relations connected in series" (p. 6). For instance, the set of numbers 46, 7, 5376, and 389 can be ordered from smallest to largest. The difference between each element is irrelevant to the ordering of the set. Also, if we represent each number as  $a$ ,  $b$ ,  $c$ ,  $d$  from smallest to largest, we know, via transitivity, that because  $d > c > b > a$  that  $d > a$ . Also, having the ordering structure enables one to know that  $b$  is simultaneously  $> a$  and  $< c$ . The "pre-ordering" child cannot coordinate these two, asymmetrical relations simultaneously (Ginsburg &

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families to 10 and be working on teen number combinations; be comfortable with operations of addition and subtraction (able to tell and show addition and subtraction stories); be comfortable with the symbolic level; be able to count into the double digit numbers.

Opper, 1988). (The task used in the present study focused on the ordering of length rather than number.)

D. R. Phillips (1991) stated:

It is crucial that the child develop mental structures of order or seriation, since these ideas are necessary for multitudes of activities in everyday life. The child's understanding of number hinges upon the development of order. As we count, "1, 2, 3, 4, 5, . . ." we know that each number is one larger than the previous number. We also know that 3 is at the same time larger than 2 and smaller than 4. Each number has its own particular position in the series of all numbers. According to Piaget, not until this aspect of ordinality (the position of a number in a series) is coordinated with cardinality (how many objects the number represents) can the child develop a concept of number. (p. 146)

It is this unique coordination, this fusion of two structures—class inclusion and seriation—that, according to Piaget (1965), constitutes the foundation for a deep understanding of whole numbers:

In our view, logical and arithmetical operations therefore constitute a single system that is psychologically natural, the second resulting from the generalization and fusion of the first, under the two complementary headings of inclusion of classes and seriation of relations, quality being disregarded. When the child applies this operational system to sets that are defined by the qualities of the elements, he is compelled to consider separately classes (which depend on the qualitative equivalence of elements) and asymmetrical relations (which express the serial differences). Hence, the dualism of logic of classes [based on likenesses] and logic of asymmetrical relations [based on differences]. But

when the same system is applied to sets irrespective of their qualities, the fusion of inclusion and seriation of the elements into a single operational totality takes place, and this totality constitutes the sequence of whole numbers, which are indissociably cardinal and ordinal. (p. viii)

#### Conservation of Number:

Piaget (1965) described three stages of number acquisition. Conservation of number is acquired during the last stage. An understanding of number requires a simultaneous coordination of its cardinal nature (how many) and its ordinal nature (the place it takes in the series of all whole numbers). The "conservation of number" task in the present study assessed only the child's understanding of the cardinal nature of number, that is, the conservation of "how many" or, the constancy of quantity. It did not deal with the ordinal nature of number.

Piaget's conservation tasks are amongst the most widely investigated and reported. A classical conservation task involves the child's first establishing equivalence between the amount of liquid in two identical, cylindrical containers. Then one amount is poured into a container that is either narrower or wider than the first container and the child is asked whether the amount is the same, more, or less. A non-conserving child will report, for example, that the amount is greater because the height of the liquid has increased. For the child, "taller" means "more." A conserving child will respond with something like, "They are the same because you haven't added or taken away any liquid, and even though the liquid is higher, the container is also skinnier" which signifies that he/ she can simultaneously coordinate two dimensions: the height and the diameter of a cylinder (Piaget, 1965, pp. 3-24).

In a conservation of number task, the child establishes equivalence between two opposite rows of objects, based on one-to-one correspondence. Then, after the objects in one of the rows are spread way out or clustered together the child is asked

whether there are more, less, or the same amount of objects. The non-conserving child will report, for example, that there are more objects in the spread-out row because it is longer. For the child, "longer" means "more." The conserving child will respond with something like, "They are the same because you didn't add or take any away, and even though the row is longer the space between the objects is also greater" which signifies that he/ she can simultaneously coordinate the two dimensions of length and density (Piaget, 1965, pp. 74-85).

During the first stage of its development, the child's understanding of number can begin through experiences with "perceptual" or "figural" numbers from 1 to 5. Counting, manipulating objects and carrying out activities with sets and configurations representing "perceptual numbers" can facilitate the development of class inclusion and seriation, and vice versa. Hence, having children working with numbers does not need to wait until class inclusion, seriation, and conservation of number have been completely attained. On the contrary, as Piaget (1972) pointed out, there is a certain "concrecence" or "growing together" of these three structures:

This does not mean . . . that the synthesis of number occurs after the structures of classification and seriation are completed, for from the pre-operational level onwards there occur figural numbers without conservation of the total; and the construction of number can facilitate that of the inclusion of classes as much as or sometimes more than the inverse. . . . There can be . . . variable collateral relationships between the three fundamental structures of classes, relations and numbers. (p. 39)

During this first stage the child has only a sense of "more" or "less" based on his/ her perception of one particular dimension such as height, width, length, general surface area, etc. "longer," "higher," etc. means "bigger" and "bigger" means "more"



even though the substance, while growing taller, has also grown skinnier. Both "increasing tallness," for example, and "increasing skinniness," i.e., "decreasing width," cannot be coordinated simultaneously (Piaget, 1965, pp. 5-13). The child is "percept bound" and "centers" on only one dimension at a time. Piaget (1965) explained:

At the level of the first stage, quantity is therefore no more than asymmetrical relations between qualities, i.e., comparisons of the type 'more' or 'less' contrived in judgments such as 'it's higher/ 'not so wide,' etc. These relations depend on perception, and are not as yet relations in the true sense, since they cannot be coordinated one with another in additive or multiplicative operations . . . . (p. 5)

The child behaves as though he had no notion of a multi-dimensional quantity and could only reason with respect to one dimension at a time without coordinating it with others. (p. 10)

The second stage of number conservation as described by Piaget (1965, pp. 79-82) is transitional. The child vacillates between what his/ her perceptions are telling him/ her, e.g., "It sure looks like more." and what his/ her logical sense is trying to tell him/ her, e.g., "The row is definitely longer, but there's also more space between (less density). They must be the same." The transition child might be able to conserve number (i.e., saying that the amounts are the same) when a second row is spread out, and then lose his/ her sense of conservation when the same row is condensed into a small cluster (i.e., saying that the amounts are now different).

In relation to the conservation of liquids task, Piaget (1965) commented:

At this [second] stage, the child is attempting to coordinate the perceptual relations involved and thus to transform them into true, operational relations.

Whereas the child at the first stage is satisfied that two quantities of liquid are equal [only] if the two levels are the same, irrespective of the width of the containers. The child at the second stage tries to take the two relations into account simultaneously, but without success, hesitating continually between this attempt at coordination and the influence of perceptual illusions. (p. 15)

Regarding the conservation of number task, Piaget (1965) stated:

At the second stage, the child is still on the intuitive plane. He sees that length and density of the rows are variable and he is perfectly coherent from the point of view of the perception in assuming that this variation involves variation in number, and this prevents him from attempting composition. Where he ceases to be logical, however, is in failing to grasp that in a contracted series the decrease in length carries with it increase in density. Instead of deducing that the result of composition is uncertain, he dissociates length and density and makes the mistake of assuming that the number of elements depends on one of the two only. (p. 92)

Children at the third stage of number conservation are capable of compensation (Piaget, 1965, pp. 82-85). They comprehend that, after a transformation, an increase in one dimension is compensated for by a decrease in another dimension, thereby leaving the quantity or substance unchanged. Piaget (1965) described the third stage child as follows:

Children at the third stage accept the fact that there is equivalence in spite of the distortion of one of the corresponding rows, without feeling the necessity for a return to the initial state. This means that they replace qualitative

correspondence by mathematical correspondence, and that they understand that the decrease in length is exactly compensated by the increase in density. (p. 94)

The importance that Piaget gave to the conservation of number as the foundation for having a deep understanding of number (beyond the perceptual numbers of 1 to 5 or so) is difficult to overemphasize. For him the notions of quantity, i.e., mathematical understanding, and conservation are practically one and the same. According to Piaget (1965):

The child does not first acquire the notion of quantity and then attribute constancy to it; he discovers true quantification only when he is capable of constructing wholes that are preserved. (pp. 10-11)

Piaget (1965) further explained that

a set or collection is only conceivable if it remains unchanged irrespective of the changes occurring in the relationship between the elements. For instance, the permutations of the elements in a given set do not change its value. A number is only intelligible if it remains identical with itself, whatever the distribution of the units of which it is composed. In a word, whether it be a matter of continuous or discontinuous qualities, of quantitative relations perceived in the sensible universe, or of sets and numbers conceived by thought, whether it be a matter of the child's earliest contacts with number or of the most refined axiomatizations of any intuitive system, in each and every case the conservation of something is postulated as a necessary condition for any mathematical understanding. (pp. 3-4)

## Formal-Operational Thought

According to Piaget (1972), Flavell (1963), Ginsburg and Oppen (1988), and D. G. Phillips (1996) formal-operational thought (beginning at about 11 or 12 years) has the following characteristics:

- A coherent, interrelated system of structures is developed. Whereas during the concrete-operational phase structures operate in isolation, in formal-operational thought they work together to solve problems and carry out inquiries.
- Concrete operations act on objects whereas formal operations are "operations on operations" or operations on propositions about objects.
- The formal-operational person has the ability to formulate hypotheses, to think in terms of all possible combinations in a problematic situation.

For example, if a child is given several packets of seeds and asked to find out which plants grow best under which conditions, the pre-formal-operations thinker will try a few different combinations but without an exhaustive system of combinations to guide the investigation. The formal-operational thinker, on the other hand, will eventually generate a scheme (probably at least jotted down) of a range of possibilities: different amounts of light, water; germination vs. growing conditions; various soil conditions, etc., applied to each type of seed.

- Whereas the concrete-operational person is focused on what is "real," and "the possible" constitutes only a small portion of this reality, the formal-operations person is immersed in a world of infinite "possibilities," and "reality" becomes just one, small, manifested portion of "the possible." The formal-operational person is much more able to transcend spatio-temporal limitations.
- The formal-operational thinker is able to deal with combinatorial reasoning, the INRC group (identity, negation, reciprocal, correlative), 16 binary operations, proportionality, probability, multiple compensations, correlation, multiple frames of reference, mechanical equilibrium, and certain conservation problems, all of which

contribute to truly scientific thinking.

### The Hierarchical Nature of the Levels of Thinking

The four types of thought discussed above--sensori-motor, pre-operational, concrete-operational, and formal operational--are hierarchically organized. The higher levels subsume the lower levels. According to Piaget and Inhelder (1969), each level attained lays the solid, necessary foundation for the next level:

Each [of the periods or stages and their sub-periods or sub-stages] results from the preceding one, integrating it as a subordinate structure, and prepares for the subsequent one, into which it is sooner or later itself integrated. (p. 153)

Because one level builds on the next, the order of development, as mentioned earlier is invariant. However, the age of initiation and attainment varies considerably as can be seen in Table 1.1 (Epstein,1979) below. In fact, many adults never attain formal operational thought or they apply it only in areas of special interest or expertise (Crain, 1985, p. 112). Epstein also estimated that only 38-40 percent of American adults can think at the formal operations level (as cited in Armstrong & Wilson, 1993, p. 313).

### The Problem with the Stages Concept

The stages concept is considered by D. G. Phillips (1996) to be "detrimental to the understanding of intellectual development." It leads to the misunderstanding that when a person moves from one stage to another that he/ she abandons completely the type of thinking at the lower stage. Phillips explains that adults can think using sensori-motor schemes, pre-operational functions, concrete-operational structures, and formal-operational thought. Also, it has been found that many adults use formal operations only in certain aspects of their personal, social, or professional life. For

everyday problems they may use concrete operations and for complex problems they may employ various levels of thinking (Craine, 1985). Hence, for assessment purposes, it can only be said that a subject performed at a certain level on a certain task at a certain point in time. As a substitute for stages, Phillips recommends the terms "levels" or "types" of thinking as used above.

Table 1.1

Percentage of Individuals in Piagetian Levels of Thought					
Age (Years)	Preopera- tional	Concrete Onset	Concrete Mature	FormalOn set	Formal Mature
7	35	55	10		
8	25	55	20		
9	15	55	30		
10	12	52	35	1	
11	6	49	40	5	
12	5	32	51	12	
13	2	34	44	14	6
14	1	32	43	15	9
15	1	14	53	19	13
16	1	15	54	17	13
17	3	19	47	19	12
18	1	15	50	15	19

### Piaget in Theoretical Context

In the previous sections, Jean Piaget's basic ideas regarding development and a description of his stages were presented. This section is devoted, in a necessarily brief manner, to placing Jean Piaget's model into a historical, philosophical, and theoretical

context.

There are several ways of organizing theoretical schools of thought.<sup>8</sup> The following scheme is based on Lawrence Kohlberg's (1987) description of three historical streams of educational ideology: Romanticism, Cultural Transmission, and Progressivism.

Jean Jacques Rousseau is viewed as the founder of Romanticism which emphasized the innate goodness of nature and the past. As applied to human development, the "child" is the "past" of the adult, hence, Romantics emphasize the inherent goodness of children's nature which will unfold, or mature, naturally if children can be isolated from the detrimental, socializing influences of western civilization. Current representatives of Romanticism, according to Kohlberg, are the followers of Freud and Gesell. An example of the implementation of the Romantic ideology can be found in A. S. Neill's Summerhill school. The "deschooling of society" concept of Ivan Illich and G. Stanley Hall (Kohlberg, 1987, p. 47) also ring of Romanticism. Crain (1985) placed Schachtel and Werner<sup>9</sup> in the Romantic school. Wadsworth (1984) also placed maturationists such as Montessori in this stream of educational ideology. A "natural" and even "rich" and "structured" environment is provided, but development is,

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<sup>8</sup> A fascinating alternative contextualization of Piaget is offered by William E. Doll, Jr. (1993) in his book, *A Post-Modern Perspective on Curriculum*, in which he included a chapter on "Piaget and Living Systems." Briefly, he considered Piaget's work as being *post-modern* and organismic, especially in its incorporation of the concept of self-organization, rather than *modern* and mechanistic, i.e., ignoring the internal purposes and will of the developing organism and its communication/ relation with the environment. Using a general systems framework, Doll viewed Piaget as transcending the open systems versus closed systems dichotomy by offering, along with Dewey, a third alternative which "provides a new level of complexity with openness and closure embedded within each other" (p. 59).

<sup>9</sup> Wadsworth (1984) places Heinz Werner in the Progressive school. This author personally favors this placement.

in the terms of Piaget (1972), completely "endogenous," "innate," and "a *priori*" (p. 19). Noddings (1995) viewed this emphasis on the innate as having its roots in rationalism. Hence, for Romanticists, educational intervention is not the focus.

Cultural Transmission has its roots in the classical empiricism of Locke, Hume, and Herbert Spencer and is grounded in associationism (Piaget, 1972, p. 10). Empiricists view humans as "blank slates" written on by the environment. Behavioral responses are learned by being "associated" with certain stimuli. The scheme is basically mechanistic with the environment providing "inputs" and behavior manifesting "outputs" (Wadsworth, 1984). Great faith is placed in science to perfect the environment in order to enhance human perfection (Kohlberg, 1987). Cultural Transmission is the predominant paradigm of Western education, especially in North America. Contributors to this approach are Ivan Pavlov, John Watson, A. H. Thorndike, B. F. Skinner, Bereiter, and Engleman (Wadsworth, 1984).

Progressivism is equated by Wadsworth with the cognitive development movement which developed as part of the pragmatic functional-genetic philosophies at the turn of the century. It views development as being the consequence of the interaction between maturation (including the genetic endowment) and the environment. The approach is more holistic and organismic because it moves beyond the "heredity vs. environment"/ "nature vs. nurture" dichotomies by focusing on the interaction between the two. This stream of ideology can be traced from Plato to Kant and Hegel and was promoted in this century by Henri Bergson, Charles Pierce, Alfred North Whitehead, and John Dewey and more recently by Abraham Maslow, Heinz Werner, Jean Piaget, Lawrence Kohlberg, David Elkind, Daniel C. Jordan, Donald T. Streets, Rheta DeVries, Constance Kamii, George Forman, David Weikart, Ed Labinowicz, Darrell G. Phillips, Dale R. Phillips, and others, granted, with a great deal of differences within what can be broadly termed (after Wadsworth) "an interactionist viewpoint" (Craine, 1985; Elkind, 1989; Jordan, 1978, 1979, 1981; Jordan &



Kalinowski, 1973; Jordan & Shepard, 1972; Jordan & Streets, 1973a, 1973b; Kohlberg, 1987; Labinowicz, 1985; Noddings, 1995; Roopnarine & Johnson, 1993, Wadsworth, 1984; Waite, 1975).

Piaget referred to his progressivist-oriented stance as constructionism and he traced his theoretical roots to Kant who posited a resolution of the idealist notion that all knowledge is a mental construction with the empiricist position that all knowledge is a copy of an externally existing world by arguing that the mind provides the "categories" of knowing ("structures" for Piaget) while the actual world provides the content. Knowledge is thus the constructed product of the interaction between the mind and the world (Elkind, 1989).

Piaget took Kant's theory further by positing that the "categories" (or, structures) are not constant but rather that they change and develop over time. His neo-Kantian contribution is that the child's knowledge of reality is very different from that of the adult's because the structures of the mind are different.

Piaget (1972) placed his own work between the extremes of empiricism and innatism and he tried to find common, epistemological ground amongst the three ideologies in an attempt to provide "an answer to the as yet unresolved question of the way in which cognition initially develops" (p. 19). He further stated:

Limiting oneself to classical statements of the problem, one can only ask whether all cognitive information has its source in objects, so that the subject is instructed by what is outside him, as traditional empiricism assumed; or whether, on the contrary, the subject possesses from the start endogenous structures which it imposes on objects, as is maintained by the varieties of *a priorism* or innatism. But even if we allow for the variety of positions between these two extremes . . . there seems to be a common postulate of accepted epistemologies, viz. the assumption that there exists at all levels a subject aware of its powers in various

degrees . . . ; that there are objects existing as such for the subject . . . ; and above all intermediaries (perceptions or concepts) which mediate between the subject and objects and vice versa, (p. 19)

In an attempt to relate his approach to stimulus-response theory, Piaget and Inhelder (1969), as interactionists, posited that the uni-directional arrow representing the direction of causality between the stimulus and the response ( $S \rightarrow R$ ) should be bi-directional ( $S \leftrightarrow R$ ) in order to express the reciprocal action which modifies both the input (S) as it is assimilated and the mental structure (R):

The input, the stimulus, is filtered through a structure that consists of the action-schemes (or, at a higher level, the operations of thought), which in turn are modified and enriched when the subject's behavioral repertoire is accommodated to the demands of reality. The filtering or modification of input is called *assimilation*; the modification of internal schemes to fit reality is called *accommodation*. (p. 6)<sup>10</sup>

It is precisely the importance given to the development of these internal schemes (or, mental structures), in this author's opinion, which distinguishes many of the constructivist approaches presently being implemented in the field of education. As was discussed earlier, the constructivist movement was founded on the work of Piaget, but there are constructivists who refer to the construction of knowledge without also

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<sup>10</sup> In order to incorporate action schemes and operations of thought into S-R theory, in this author's opinion, it would be necessary to insert into the formula the mental structures (MS) as mediators (or filters) between the incoming stimulus and the outgoing response (behavior):  $S \leftrightarrow MS \leftrightarrow R$ , the latter two of which would be modified in a reciprocal, cybernetic fashion.

concerning themselves with the reciprocal construction of structures (Forman, 1993). To be truly Piagetian, however, an approach would need to at least balance the two, if not emphasize the development of structures. Darrell G. Phillips (1996), upon whose work the present study was based, referred to himself as a "structuralist" rather than a "constructionist," in this author's opinion, in order to clarify this difference. The necessary complementarity of the two was pointed out by Piaget (1972):

The achievement of knowledge is . . . to be explained in terms of a theory indissolubly linking structuralism with constructivism, every structure being the resultant of a genesis and every genesis being the transition from a more to a less elementary (or more complex) structure. (p. 12)

### Statement of the Problem and the Research Questions

The concerns of the present study were: (1) to ascertain whether or not there are differences of performance on the targeted tasks amongst children of the same age range; (2) to determine whether or not there are differences of performance on the tasks amongst the three different age ranges: 5 years, 6 months to 6 years, 5 months (5.6-6.5); 6 years, 6 months to 7 years, 5 months (6.6-7.5); and 7 years, 6 months to 8 years, 6 months (7.6-8.6); and (3) to find out whether or not there is a difference in task performance between males and females.

Hence, the principal research questions were:

- I. Is there a difference in task performance amongst subjects of the same age range?
- II. Is there a difference in task performance amongst different age ranges?
- III. Is there a difference in task performance between males and females?

In conclusion, this introductory section has justified the present study on the grounds of the importance of investigating further the development of mental structures needed for classification, ordering and the conservation of number, and of widening the international research base of studies utilizing the standardized tasks and protocols developed by D. G. Phillips and his colleagues. The basic ideas of Piaget regarding cognitive development were reviewed: his identification of maturity, social transmission, experience, and equilibration as the four interacting factors which influence development; his focus on the investigation of children's gradual attainment of intellectual structures; his understanding of intelligence as consisting of the mental structures used to construct knowledge; his notion of logical structures as systems of transformation used to act upon, manipulate, and make sense of data; his proposition that organization and adaptation comprise the mechanism which enables humans to integrate lower order processes into higher order processes; and his differentiation of intellectual development into four broad, hierarchically organized stages of development--sensorimotor, pre-operational, concrete operational, and formal operational thought. A more detailed description was provided of the four mental structures involved in the present study--graphic and non-graphic collections, class inclusion, ordering by length, and conservation of number. The problem was discussed regarding the misinterpretation of Piaget's stages of thinking as being sequentially organized which results in the misconception that a person abandons one type of thinking as he/ she gains competence at higher stages, rather than the more accurate view of hierarchically-nested stages in which, as a person attains higher stages of thinking he/ she still has access to and utilizes lower-level mental structures in particular situations. Piaget's work was placed within a Kantian, progressivist, constructivist philosophical/ theoretical context. And finally the principal research problem and questions of the present study were delineated.

## II. REVIEW OF THE RELEVANT RESEARCH LITERATURE

Cohen (1983, p. 68), citing data from *The American Psychologist* yearly index of total references to particular authors, stated that the Piagetian model is the most widely researched paradigm in psychology after that of Freud. For example, in a single year (1978) Piaget received 1,071 citations compared to 1,479 for Freud. Cohen referred to a study which indicated that between 1950 and 1980 there were over 2000 experiments and studies whose principal aim was a variation on Piaget's ideas. Although the Piagetian "fever" has subsided since 1980, research continues, as attested by the present study and others cited in the following review and in Section VI which summarizes the critiques and assessments of Piaget's work.

Because Piaget's work has been discussed broadly in the prior pages, this section will focus on the review of the research literature which relates to each of the four tasks under study: collections, class inclusion, ordering, and conservation of number. For each task the literature review is comprised of three general categories: the research of Piaget and Inhelder, the studies of the D. G. Phillips research group, and the pertinent investigations of other individuals and research groups. This categorization was devised to reflect the degree of similarity between this present study and other studies. The studies carried out by Piaget and Inhelder dealt with the same structures, but the materials and protocol were not necessarily the same as in the present study. The cited studies by D. G. Phillips and his research group focused on the same mental structures and, for the most part, used the same materials and protocol as in the present study. Therefore, they are of greater relevance. Other pertinent studies defined the structures somewhat differently and they used different materials and protocols. Nevertheless, the findings are of relevance to this inquiry. Some of the studies investigated more than one mental structure. These will be reviewed in the section which this author determined to be the most relevant.

## Research on Collections

### Piaget and Inhelder's Investigations Regarding Collections

Inhelder and Piaget (1969) conducted a study of graphic and non-graphic collections, various types of classificatory structures, and seriation. The investigation involved 2,159 subjects aged 4 to 13 years. The number of subjects participating in the collections assessments was not indicated. As in all of Piaget's reports of his studies, there was no mention of gender-related differences in performance. The material used consisted of vary-colored squares, triangular shapes, rings, and half-rings, made of wood or plastic. The most usual protocol instruction was, "Put together things that are alike," which was frequently extended with the instruction, "Put them so that they're all the same," "Put them so that they're just like one another," or "Put them here if they're the same, and then over there if they're another [sic] lot different from this one but the same as each other" (p. 21). Data consisted only of interview transcripts. The percentages of subjects passing tasks were not presented. Rather, Inhelder and Piaget limited themselves to presenting examples of the various stages in the development of collection ability, which they summarize as follows:

The process consists of starting with a large number of small juxtaposed collections, and gradually grouping them together through a series of comparisons involving both retroaction and partial anticipation, until one obtains a few large collections differentiated into sub-collections. (p. 58)

These same stages were observed in the present study.

## The D. G. Phillips Group's Investigations Regarding Collections

D. G. Phillips (1989) carried out two Collections Task studies as part of a three-year, longitudinal study using the same set of paper shapes and the same protocol used in the present study. In 1983, a group of investigators directed by Phillips assessed 342 entering kindergartners enrolled in traditional classrooms. The following year the same group, now in first grade, was tested again. In order to preserve consistency, data were taken only from the control group (113 subjects), i.e., those who were in traditional classrooms and not in the experimental group which engaged in activities aimed at developing classificatory structures. The data for the first graders are presented in Table II.1 below (D. G. Phillips, 1996, p. 45). These research studies are of particular interest because the protocols, materials, and scoring criteria were the same as those used in the present study.

Table II.1

Collections Task Data--% in Each Response Category							
Grade	N	G1-G6	NG1	NG2	NG3	NG4	Sig. Gender Difference
K	342	35%	19%	20%	22%	3%	No
1	113	18%	25%	27%	29%	2%	No

In relation to the present study, what is of interest to note is not the exact percentages but rather the evidence that there are differences of performance amongst children of the same age and from one age to another.

### Other Pertinent Research Regarding Collections

Raven (1970) trained a group of second and third grade students in Piagetian classification tasks ranging from collections to class inclusion. The logical groupings

were broken down into twelve rules which were presented to children in exercise workbooks. It was found that the third grade students trained in classification achieved higher scores on the assessment tasks than did a control group which received no training in classification. What was not discussed was whether or not the benefits of this training transferred to everyday and academic situations requiring classification structures.

Because many of the issues concerning collections tasks are related to class inclusion, other related studies are reviewed in the following section. (For a summary of the critiques of the Piagetian Collections Task see the sections, "Class Inclusion" and "Collections," in Section VI.)

### Research on Class Inclusion

#### Piaget and Inhelder's Investigations Regarding Class Inclusion

In 1941 Jean Piaget published *The Child's Conception of Number* (1965) which contains a description of a number of studies concerning class inclusion. Class inclusion was included in his investigations because he "considered number as a seriated class, i.e., as the product of class and asymmetrical relation" (1969, p. 161). But when he said "product" he did not mean that seriation and classification need to be fully formed first. Rather, he saw them as developing in a mutually dependent, intertwined way: "Instead of deriving number from class, or the converse, or considering the two as radically independent, we can regard them as complementary, and as developing side by side, although directed towards different ends" (p. 161).

The studies presented in *The Child's Conception of Number* appear to be exploratory. The number of subjects was not given. Materials and protocols varied greatly. With a group of 4- to 6-year-old children Piaget used a set of several brown and



two white, wooden beads. Three stages of attainment were found. The following is an example of a Stage I clinical interview:

Stro (6; 0 [age]): 'Are there more wooden beads or more brown beads in this box?—More *brown* ones.—Why? *Because there are only two wooden ones.*--But aren't the brown ones made of wood?--*Oh yes!*--Well then, are there more brown ones or more wooden ones?—*More brown ones.*' (p. 164)

Because so many children gave similar answers Piaget became concerned about the nature of the task. It is fascinating to observe, in the evolution of his methods, how much he sympathized with the children and how hard he struggled to design tasks which would enable them to demonstrate the structure, if indeed, they had it. In the bead task, for example, he was concerned that both the superordinate and subordinate classes contained the same name--"beads": "The difficulty seems to be increased by the fact that there is no single word to indicate the general class and the particular classes, but only combinations of words, 'wooden beads', 'brown beads', 'white beads', in each of which 'beads' occurs" (p. 166). This led him to experiment with other sets of materials, for example: flowers, poppies, and bluebells; children, girls, and boys; things, cones (roofs), and beads; and blue beads, square blue beads, and round blue beads. When he obtained better results with the children-girls-boys categories, he attributed it to the greater clarity of having distinct class and subclass names.

Piaget also thought that perhaps the wrong answers were due to the great disparity between the number of brown beads (many) and the number of white beads (two); that perhaps perceptually the difference in number was too overwhelming, thereby creating a "systematic illusion in the child's mind" (p. 166). Therefore, he tried sets with 20 brown and 18 green beads; and 10 big yellow beads and 15 small yellow beads. The results were about the same: children under 7 or 8 have "systematic

difficulty . . . in including one class in another, and in understanding that a total class is wider than one that is included in it" (p. 166). However, when he used sets with an identical number of brown and white beads there was an increase in correct answers. About half of the 6-year-olds and "even some who were only 5" were able to answer correctly. This improvement occurred because, he believed, the subject "can consider simultaneously one set from the point of view of the brown beads and the other from that of the whole set" (p. 170).<sup>11</sup>

Stage II children were characterized by being able to answer some questions but not others and by vacillation, for example:

Tail (7; 2): 'Are there more brown beads or more wooden beads in this box?--*More brown ones.*--Are the white ones made of wood?--*Yes.*--Then are there more wooden beads or more brown ones?--*More wooden ones because there are two white ones as well.*--Which would be longer, a necklace made with the brown beads or one made with the wooden beads?--*They'd be the same.*--Are the white beads made of wood?--*Yes*--Then which necklace would be longer?—*Oh! the wooden one would be longer because there are the two white ones.*' (p. 175)

For Stage III children the discovery is spontaneous and not pre-meditated:

Laur (7; 2 . . .): 'Are there more brown beads or more round beads in the box?--*More brown ones. Oh no!* (spontaneously), *more round ones, because*

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<sup>11</sup> D. G. Phillips (1996) took into account this concern for class names by using the set--animals, cows, and horses. The present study used the set: animals, fish, and crabs. The numbers used by Phillips and in the present study were not too identical and not too disparate: 9 of one animal and 4 of the other.

*there are the two white ones as well.*—Which would longer, a necklace made with the brown ones, or a necklace made with the round ones?—*The one with the round ones.'*

Piaget attributed the success of the Stage III children to "reversibility"; i.e., their ability to see the general class, destroy it by breaking it into sub-groups, then recreating it (reversibility) in the mind while comparing it to one of the subsets to determine which contains more.

In 1959 Inhelder and Piaget published *The Early Growth of Logic in the Child* (1969) which contains several studies concerning class inclusion. Unlike the earlier work, the number of subjects at each level was given along with the percentage of subjects passing the tasks. Also, the tasks shifted to using only materials which have distinct class and sub-class names; and the protocol questions asked of each subject were more uniform--while allowing for the ample use of "pursuit" questions.

In one experiment 20 pictures were used representing 4 colored objects and 16 flowers: 4 yellow primulas, 4 vary-colored primulas, and 8 flowers of other varieties. After allowing subjects to spontaneously classify the pictures, questions were asked in order to establish that the subject can identify the class and subclasses. Other questions were asked about the taking away of a class or subclass: "If all the flowers are picked, are any primroses left?" and "If all the primroses are picked, are any flowers left?" But the telltale questions, the ones that require reversibility and the conservation of the class while comparing it quantitatively to a sub-class were: "Are there more primulas or more yellow primulas in this bunch?" and "Are there more flowers or more primulas?" A few of the typical Stage I replies follow:

Fav (5; 4) . . . . "Are there more primulas or more yellow primulas?--*More primulas.*--And are there more primulas or more flowers among all these?--*More*

*primulas."*

Ter (5; 8) . . . . " . . . more yellow primulas or more primulas?--*No, there are more yellow primulas.*--. . . more primulas or more flowers?--*More flowers* (but he points to A' [subset of primroses] and not to A + A' [set of flowers])"<sup>12</sup>

Ric (6; 6) ... . "How would I be making a bigger bunch, by taking the primulas or by taking the yellow primulas?--*They're both the same.*--And a bunch of flowers or a bunch of primulas?—*It's the same.*"<sup>13</sup> (pp. 102-103)

Again, Inhelder and Piaget divided into three stages the acquisition of the class inclusion structure and emphasized the ability of the Stage III child to maintain the identity of the superordinate class so that it continues to exist even while its component, subordinate classes are separated from it in thought.

It is also of interest to note that Inhelder and Piaget recognized that a Stage II child may be "intuitively aware that the whole is the union of its parts and that one part is distinct from another, even though he cannot compare the extensions of the part to the

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<sup>12</sup> This example was especially chosen in order to point out one of the advantages of using clinical interviews which involve the manipulation of objects over the use of written or exclusively verbal assessments. In this case, the response would have been considered correct, but the child's action betrayed his lack of the class inclusion structure.

<sup>13</sup> Besides illustrative purposes, I included a number of examples of class inclusion task responses for historical reasons which continue to fascinate me (and I hope the reader). It is my understanding that it was these same types of illogical responses to this same type of class inclusion task which originally captured the interest of both Piaget while working in Paris at the Binet Laboratory in Paris and Darrell Phillips while observing his daughter participate in a university study in Florida. Such a small phenomenon provoked sixty years of research in Piaget's case and over thirty years in Phillips' (Phillips, 1995). How fortunate we are to have benefited so much from the sensitive observations of children's mistakes!

whole" (p. 106). This intuitive knowledge appears to depend on the amount of active experience (physical and/ or mental) that the child has had with the particular concrete objects used in a task.

Table II.2 (Piaget, 1969, p. 109) shows the percentages of correct answers to both key questions (more primulas or yellow primulas? more flowers or primulas?) amongst 69 subjects, ages from 5-10 years. No data are given for difference in performance between boys and girls.

Table II.2

% of correct answers to questions on inclusions of classes of flowers				
Number of Subjects:	20	19	17	13
Ages:	5-6	7	8	9-10
Percentage correct:	24	26	61	73

In relation to the present study, what is of interest to note is not the exact percentages but rather the evidence that there are differences of performance amongst children of the same age and from one age to another. Also note that even as late as 9-10 years (about fourth grade) 27% of the children have still not formed the class inclusion structure.

In a subsequent study Inhelder and Piaget (1969) used pictures of animals for a class inclusion task: 3-4 ducks, 3-5 other birds (cock, sparrow, parrot, etc.), and five animals other than birds (snake, mouse, fish, horse, poodle). Participating in the study were 117 children aged 7 to 13 or 14. Key questions included: "Are there more hens or more birds in the tray?" "Are there more poultry or more animals in the world?" "Are there more animals that fly in the world or more birds?" "Are there more birds or more animals in this box?" (pp. 112-116). The results are shown in Table II.3 below.

Table II.3

% of correct answers to questions on inclusions of classes of animals					
Number of subjects:	17	22	14	17	47
Ages:	8	9	10	11	12-13
Percentage correct:	25	27	42	46	67

Inhelder and Piaget expressed surprise at the lower performance in comparison with the flower task. For example, 61% of the 8-year-olds passed the flower task whereas only 25% of them passed the animal cards task. The investigators explained: that this "must be due to the fact that these [animal] classes are more remote from everyday experience and therefore more abstract"; that "a child cannot say that ducks are birds and birds are animals by simply relying on experience drawn from his own actions, as he can for squares and circles which he has drawn and for flowers which he has picked; that the child "is compelled to rely far more on purely linguistic concepts. . ."; that "the emergence of concrete operational reasoning depends very closely on the intuitive character of its content" (p. 106); that "there is no formal mechanism underlying this sort of classification"; and that "that is why we call it a concrete operation--the level of reasoning varies with the character of the content to which it applies" (p. 114).<sup>14</sup>

Comparing the two protocols above and judging from this author's own experience with this type of task, it could be hypothesized that the addition of the term "in the world" in several of the animal-task interviews could cause confusion amongst the children because they no longer can rely solely on the objects within their perceptual field. This was not the case with Piaget's flowers task. To test this hypothesis the "in the world" statement would need to be added to the flowers-protocol and the results compared.

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<sup>14</sup> These insights of Inhelder and Piaget were included because they are pertinent to the criticisms of the Piagetian model which will be discussed in Section VI.

Piaget and Inhelder further pointed out that they believe firmly that success on these tasks was "not a question of mere verbal facility"; that class inclusion "has not been acquired merely because the child talks correctly and uses verbal concepts which reflect the inclusions implicit in the language of adults"; and that class inclusion "is a genuine logical operation" which forms "the basis of any classification which really does order classes" (pp.117-18).

#### The D. G. Phillips Group's Investigations Regarding Class Inclusion

D. G. Phillips (1996, p. 79) reported eight Class Inclusion Task studies which he and his collaborators carried out. The percentage of subjects passing the task for each grade level is presented in Table II.4 below. (The age ranges of the students in the present study corresponded roughly to kindergarten, first grade, and second grade.) The studies which used protocol, materials, and scoring categories most like the ones used in the present study were D, E, and F. No significant gender differences in performance were found.

#### Other Pertinent Research Regarding Class Inclusion

Mwamwenda (1985), in a study of 178 Canadian children between the ages of 5 and 7, reported that class inclusion is generally easier than and emerges before conservation of liquid. This author believes that a possible explanation for the relative easiness of the class inclusion task could have been the nature of the task materials. Mwamwenda used two sets of objects. The first set contained five pieces of plastic fruit--two apples and three bananas. The number 5 is a perceptual number and the difference between the two sub-sets was only 2 (a ratio of 1.7:1). These two factors together could have made it easier for the subjects to recognize the superordinate set

Table II.4

Class Inclusion Task: % Passing				
Study	Grade	n	% Passing	Sig. Gender Differences
A	4	36	22%	no
	5	36	58%	
	6	36	42%	
B	2	30	70%	no
	5	30	97%	
C	1	34	53%	no
	2	34	65%	
	3	34	88%	
D	PreK	28	29%	no
	K	36	25%	
	2	36	69%	
E	2	97	33%	no
F	91	91	37%	no
G	K	34	12%	no
	1	36	28%	
	2	32	50%	
H	3	28	57%	no
	6	25	68%	
	9	26	92%	
	12	25	88%	

of fruit. In the second set there were eight plastic animals—three horses and five cows. The total was greater but the difference was still only 2 (a ratio of 1.6:1). It could be that not having one sub-set which was significantly greater than the other was not enough of a perceptual "temptation" to say that there were more cows than animals. In the



present study, on the other hand, there were several more animals--13 in all--and the difference in number between the two subsets was also greater--9 crabs and 4 fish (a ratio of 2.3:1). With this set of materials the group of 9 crabs really could have looked like "a lot" to the child. Thus, the task in the present study could have been more challenging than those used by Mwanwenda.

Several studies focused on the performance of subjects on class inclusion tasks as a function of language rather than a function of logical ability. Markman and Seibert (1976) administered two forms of a Class Inclusion Task in three related studies with kindergarten and first grade students. One question form used "collective nouns" such as "family," "bunch," "pile," and "class." The other form used corresponding "class questions" which contained the words "frogs," "grapes," "blocks," or "children." They found a significantly greater number of correct responses to "collections questions" such as, "Who would have more pets, someone who owned the baby frogs or someone who owned the family?" than to "class questions" such as, "Who would have more pets, someone who owned the baby frogs or someone who owned the frogs?" They concluded that "collections form psychologically more coherent wholes than classes" (p. 566).

There are two problems with this study--the definition of "collections" and the key experimental question. From a Piagetian viewpoint and based on the children's responses in the present study, the "collections questions" were referring only to a sub-class of collections which happened to have a "collective noun" name. In the present study, when children were asked to name the collections they created, for graphic collections they named the objects--"tire," "tower," "house," etc., and for non-graphic they named the contents of the piles--"the triangles," "the half-circles," etc. They did not use collective nouns such as "the *pile* of squares" or "the *bunch* of rings." Hence, "collective noun" collections are a very limited type of collection. The authors gave them importance, however, because they seemed to facilitate class inclusion.

The other problem was the use of terminology. On three of the four control group questions the superordinate and subordinate classes included the same word: "frogs" with "baby frogs," "grapes" with "green grapes," and "blocks," with "blue blocks." The only exception was the combination of "children" with "boys." The collections questions, on the other hand, used distinctive noun combinations: family/ baby frogs, bunch/ grapes, pile/ blocks, and class/ boys. D. G. Phillips (1996) pointed out that class and sub-class terms containing the same word can cause confusion in children. For this very reason he altered Piaget's original Class Inclusion Task which referred to wooden beads and brown beads. Instead, he recommended other objects with class-subclass terms such as "animals" and "cows" or "fruit" and "bananas." Thus, the use of same-term referents could have lowered the percentage of possible successes amongst the subjects answering the "class questions." This same mistake was repeated in the second experiment and in the third, which used distinctive whole-part terms such as "pig" and "body" (versus "head") or "butterfly" and "wings." The results were nearly the same as those of the "collections questions." Again, the better results could be attributed to the use of distinct class-subclass terms. Markman made this same mistake in a previous study in which she obtained similar results by comparing daisies with white daisies, and balls with blue balls (1973). In a later study (1978) this weakness was partially corrected. For "class questions" she intentionally created half of the questions using a superordinate term plus an adjective (e.g., blocks and blue blocks) and the other half using distinct terms (e.g., children and boys). (Unfortunately the data for these two sub-groups were not included in the report.) However, in the "collections questions" all of her class/ sub-class terms continued to be distinctive (e.g., army and soldiers, forest and trees, band and musicians, team and players, crowd and people). To correct this flaw all four "collections questions" would need to be of the "children-boys" type. In the present study, for example, if the influence of collection questions had been investigated, a subgroup could have been given the Class

Inclusion Task using the terms "family" and "baby crabs" (versus "mommy and daddy crabs") instead of "animals," "crabs," and "fish."

Fuson and Lyons (1988) replicated Markman's class inclusion experiment discussed above. Unfortunately, they based their procedure on the 1973 and 1976 studies instead of the 1978 study. Hence, they repeated the same flaw by comparing "piles" with "blue blocks" and "blue blocks" with "blocks." Their findings concurred with Markman's.

Hodkin (1987) also questioned the Markman finding and recommended submitting the data to a "performance model analysis" which is a method for factoring out the amount of guessing taking place in correct and incorrect answers.

In the Hodkin (1987) study the role of language was further pursued by comparing instructions using verbal labels, e.g., "These are triangles," with instructions with "reduced language" using visual symbols made of cardboard shapes cut larger than the task objects and outlined in corresponding colors. The instructions did not include names for the objects (e.g., "These are ones like this [triangle symbol indicated]). Contrary to expectations the "reduced language" condition did not produce better performance on the class inclusion task. Rather, it produced more guessing than the standard Piagetian protocol.

In a previous study Hodkin (1981) found that by including the modifier "all" to the superordinate term, 3- to 8-year-olds gave more correct answers, e.g., "Are there more smarties or more of all the candies?" Hodkin concluded that class inclusion develops in children at a younger age than found in those studies which use the standard Piagetian question, "Are there more smarties or more candies?" From a Piagetian point of view, however, it must be kept in mind that the use of more natural or appropriate language alone is not sufficient to bring into being a mental structure sooner. For the children in this study who gave correct answers, it would be useful to examine their construction of class inclusion within the context of other cognitive abilities. For example, their class

inclusion ability would be brought into doubt if they could still not pass the Collections Task or if they had not yet developed the beginning topological structures which usually appear before class inclusion (D. G. Phillips, 1996).

Caracciolo, Moderato, and Perini (1988) reported that in a series of experiments they obtained significant improvement of performance on tasks of class inclusion, conservation of number and length, and categorization, by manipulating perceptual cues (e.g., for a conservation of length task, ruled cards were used to cue the equal length of two pencils, especially after one was moved), verbal rules (e.g., for class inclusion, "The blocks may be blue or red, but they are all wooden blocks"), and training (i.e., a variety of tasks with feedback). The authors, operating from an "interbehavioral model" as opposed to a structuralist model, interpreted their findings as showing that the occurrence of complex cognitive behaviors is more sensitive to specific perceptual cues, linguistic evolution, and training than to general structural deficits. The question that the authors did not address is why some subjects are able to pass the same tasks without perceptual cues, verbal rules, or training. The doubt is raised as to whether or not they were actually testing the structures. Perhaps they were assessing the children's ability to read perceptual cues, apply verbal rules, and implement training in only situations which are very similar to the training experience. This is the problem with a behaviorist approach: operational-like behavior can be elicited without the child having developed the mental structure which enables him/ her to function without the need for perceptual cues, rules, and training, that is, functioning with logic as the basis of the operational behavior.

In another language-related study, T. S. Mwamwenda and B. B. Mwamwenda (1989) investigated how Botswana children in Southern Africa would perform on class inclusion and other tasks when judgement versus judgement-plus-explanation was used as the criterion. The subjects' rate of success was significantly higher in the judgement-only tasks. The authors offered several explanations which various

researchers have pointed out: that in some African cultures asking for an explanation after an answer has been given is not congruent with normal verbal interactions; that in some cultures neither adults nor children are used to expressing their thoughts verbally; that in one study Algerian children, when engaged in Piagetian dialogue, "tended to get rather upset"; that in some cultures, when a person asks for an answer for something when he/ she already knows the answer, the question may be interpreted as a challenge or a riddle, or that the respondent's answer is wrong which leads to a change of answer, tension, and less talk. The authors pointed out that some researchers prefer judgement-only responses while others insist on using judgements plus explanations. They admitted that not requiring explanations renders their data less comparable internationally, but they pointed out the cultural constraints on the Piagetian interview method. They concluded by recommending the use of both methods.

What the authors did not discuss is the role of chance when the judgement-only method is used. They explained what materials were used but not what question was used or the order of superordinate and subordinate terms. For example, if the question was, "Are there more oranges or more fruit?" first of all, there is a 50/50 chance of making a correct judgement, and secondly, it has been shown that children who do not know the answer to an either/ or question tend to answer with the second choice (Siegel & Goldstein, 1969). In the question above, this would be the correct answer and would, therefore, increase the number of successful performances.

In his investigation of whether or not class inclusion has mathematical prerequisites, Robert L. Campbell (1991) argued that the standard Piagetian question--"Are there more carrots or more vegetables?"--is known to be too ambiguous because it lacks the markers *all* and *only* used in ordinary speech. Therefore, he changed the question to: "Which is more, *all* of the vegetables or *only* the carrots?" The percentages of subjects passing the class inclusion task with sufficient reasons were:

30% for kindergarten, 44% for first grade, and 65% for second grade.

Regarding the question of there being mathematical prerequisites for class inclusion, Campbell found that class inclusion was a separate classificatory domain which relies on logical necessity for its solution and not on number knowledge, including exact class compensation, addition/ subtraction, additive composition, counting, and estimation.

Another area of research on class inclusion concerns its relation to initial reading. For example, Althouse (1985) compared the progress of kindergarten children in initial reading taught through a language experience approach and performance on Piagetian tasks of class inclusion, seriation, and conservation of number. Significantly high correlations were found for the class inclusion tasks ( $\alpha = .001$ ), conservation of number--transformation of spatial configuration ( $\alpha = .001$ ), and conservation of number--intentionally counting before reconfiguration ( $\alpha = .01$ ). Conservation of length and seriation did not reach a level of significance.

In a unique study which examined the correlation between successive and simultaneous information processing, Dash, Puhon, and Mahapatra (1985) administered a battery of tests to a group of 60 Indian children in the age group of 7-9 years. The series of assessment instruments included both a set of tests known to distinguish children's processing preference and a set of Piagetian tasks for class inclusion and transitivity. The investigators found that simultaneous processing was used by children to master the class inclusion concept but not the transitivity concept. This finding is different than those of a previous, similar study (Mwamwenda, Dash, & Das, 1984) which showed that children using simultaneous processing did better on class inclusion, conservation of liquid, *and* transitivity, and that success on the Class Inclusion Task was helped by both types of processing. Nevertheless, the line of research is promising. As described by the authors, the concept of simultaneous vs. successive processing is similar to global vs. analytical thinking referred to in the

learning styles literature (Dunn & Griggs, 1995). If so, the implication may be that learning style preferences could enhance or possibly even deter children's mastery of particular concrete operations.

In two studies which attempted to correlate memory ability with classification ability measured via tasks of class inclusion and hierarchical classification, Keasey, Crawford, and Eisert (1979) compared two groups of 3- to 6-year-olds--those who passed vs. those who did not pass the tasks--in their ability to recall a randomly presented series of objects. They found that the children who had formed the class inclusion and hierarchical classification structures were able to better organize the items and recall a greater number. The pre-structure children, when cued with the category names, could organize the items but they did not recall a greater number of items. The investigators concluded that activities aimed at helping children to see how information is classified (e.g., days, weeks, and months) could help them to form the classification structures which might, in turn, strengthen their encoding and retrieval abilities.

In conclusion, this literature review indicates that there are different ways of defining class inclusion, that there are several contextual and task variables which can affect the performance of children, and that, in spite of the controversies, interest continues in the investigation of why the responses of younger children are so different from those of older children and adults on class inclusion tasks. (For a summary of the critiques of the Piagetian Class Inclusion Task see Section VI.)

## Research on Ordering

### Piaget and Inhelder's Investigations Regarding Ordering

In Piaget's *The Child's Conception of Number* (1965) he described four exploratory studies of seriation (ordering) aimed at exploring three possible operations: qualitative seriation, qualitative correspondence between two seriations (similarity), and numerical correspondence (ordinal) between two series. The subjects ranged in age from 4 to 7 years but the number of subjects is not given. The materials and protocols varied considerably. No statistics were given for percentages passing the tasks. Nevertheless, the studies are of great relevance because it can be seen how the Ordering Task in the present study was developed by D. G. Phillips based on Piaget's procedures.

In the first study children were presented with ten wooden dolls of the same thickness but seriated in length and width; ten sticks, also seriated in length; and, as an alternative to the sticks, ten plasticine balls of differing sizes. The children were asked to "arrange the dolls and sticks so that each doll" could "easily find the stick that belongs to it." After two corresponding rows were made, the interviewer spread out the row of sticks or balls and condensed the row of dolls. A doll was pointed to and the question posed, "Which stick will this one take?" Following this, the order of one of the series was reversed and the same question asked. The next task was to find one of the sticks or balls corresponding to one of the dolls when one or both series of objects had been disarranged. A final test was put to the children by pointing to one of the middle dolls and asking them to find the sticks that belong to the bigger dolls and then those sticks belonging to the smaller dolls (Piaget, 1965, pp. 97-98).

Piaget found three general stages in the children's development of seriation: (1) global comparison based on attention being given to a limited set of perceptual cues,



but without exact seriation or correspondence between the two sets, (2) intuitive, progressive seriation and correspondence based on trial and error, but without reversibility, and (3) operational seriation and numerical correspondence showing reversibility. He described three sub-stages of each of these stages and gives examples of student responses for each. Piaget showed surprise at the "strange behavior" shown in some of the pre-seriation responses. For example, during the first stage, instead of ordering the sticks, the children either grouped them into sub-series of 2-4 elements which could not then be put together; or, they created a stair-case appearance of the tops of the sticks while disregarding the bottoms. During the second stage, Piaget was amazed that a child, when "merely" asked to place together the sticks and dolls that were bigger or smaller than an indicated element, so thoroughly confused ordinal and cardinal values that even the number of dolls and sticks was not equal!

In the second investigation the subjects were given a set of ten sticks of varying lengths and asked to make a series. Then nine more sticks of varying lengths were presented and the subjects were asked to insert them so that a single series could be formed ranging from shortest to longest. After this was accomplished the interviewer pointed to a stick midway up the "stairway" and asked how many steps a doll would have to climb to get there and how many steps must be climbed to get to the top. Finally, the series of sticks was disarranged and the last question was repeated. To answer it the child had to reconstruct, on his/ her own, the series.

Again, Piaget found the same range of developmental levels which he divided into the three distinct stages mentioned above. There were children who had trouble with different phases of the task. The question was raised as to why some children could make the series but not make the insertions. The explanation given was that a simple series can be made by a simple "add on" method in which the child simply searches for the next shorter (or longer) stick in the remaining set of unused sticks and

then adds it on to the one(s) he has seriated. The insertion task, on the other hand, requires the simultaneous coordination of finding out not only what stick is longer than the one to be inserted but, also, which one is shorter. Hence, for insertion, the child has two tasks. He/ she must examine and compare the lengths of the stick to be inserted (by considering both ends of the stick, and not just one end) and he/ she must examine those sticks to the left *and* to the right--a large order for many children!

Another interesting finding was that some children, when asked how many steps a doll needed to climb to get to a designated point and how many more to get to the top, would reconstruct the entire series and then count the ordered steps. But those who were able to separate the cardinal aspect of number from the ordinal aspect would reconstruct the stairway *only* to the length of the stick where the doll was to have been standing and then, to find out how many steps remained to the top, would simply count the remaining sticks in random order.

In the third experiment the children were presented with a "unit" square of cardboard and varied-length rectangles each of which was a multiple of the basic square "unit." The child was asked to make a series and then asked how many of the square cards could be made with each of the others, first in increasing order and then in random order. Later, the cards were mixed and the questioning was repeated. When a child could answer, for example, that a randomly chosen rectangle would make six squares without measuring or counting spaces with his/ her finger, but rather by reconstructing the series and counting the steps in the series to the *6th* rectangle then he/ she was considered to be able to coordinate the ordinal and cardinal values in the task.

In the fourth study of seriation described in *The Child's Conception of Number*, the materials consisted of a graduated set of seven hurdles with eight uniform mats which were to be placed "before" and "after" each hurdle so that a doll runner would not be hurt. The questioning focused on the coordination of the relation between *n* hurdles

and  $n + 1$  mats. Passing the task required the coordination of cardinal and ordinal relations. Three stages were found, similar to those reported above (Piaget, 1965).

In *The Early Growth of Logic in the Child* (1969) Piaget and Inhelder reported the results of three studies which focused on seriation. The first was actually a previously published study carried out by Inhelder and Vinh-Bang. It was a replication of the seriation task described above which used one set of sticks to be seriated and another one to be inserted in the first series. One hundred thirty-four children from 4 to 8 years of age participated in the study. Three stages were found and the percentages of each age level at each stage were presented as shown in Table II.5 below. Results showed that, for this set of objects, systematic seriation by length was not reached until 7-8 years.

Table II.5

Development of Seriation: % Scoring at Each Stage					
Age-->	4 yrs.	5 yrs.	6 yrs.	7 yrs.	8 yrs.
No. of subjects-->	15	34	32	32	21
Stages:					
IA: No attempt at seriation	53	18	7	0	0
IB: Small uncoordinated series	47	61	34	22	0
II: Success by trial-and-error	0	12	25	15	5
III: Success with operational method	0	9	34	63	95

In the second experiment that was reported, the focus was on determining whether operational seriation develops from perceptual schema or from sensori-motor schema. Eighty-eight subjects were presented with a jumbled series of colored rods.

Before making a series they were asked to make a graphic drawing of what the seriation would look like when it was completed; first, using colored crayons and then a pencil. Results were recorded for both the "anticipatory" graphic seriations and for the seriation of the objects each of which was broken down into three stages. A "global anticipation" portrayed a seriated set of lines but with errors in color coordination. "Analytic anticipation" consisted of accurate order of lengths and colors. Although it might seem that making an abstract drawing would be harder than ordering a set of concrete objects, it turned out that making the anticipatory drawings was easier, especially the pencil drawings which did not have the complicating factor of color-coordination. The explanation was that drawing only required making one line longer (or shorter) than the previous one, but ordering the objects required finding a rod that was simultaneously longer than some and shorter than others. If subjects could not seriate accurately, neither could they make an accurate drawing of a series. And, if they could draw or seriate objects accurately, they could also do the opposite accurately.

The conclusion was that systematic seriation is not based on perception but on experiences of seriation activities; perception playing an important but secondary role. In fact, perception, as shown via the drawings, was shown to improve due to the influence of successful seriations of objects (actual and mental). The results are shown in Table II.6 below.

In order to corroborate their findings, a similar but tactile-based seriation experiment was carried out with 50 subjects of the same age range. The aim was to eliminate the visual-perceptual factor. Sticks were handled behind a blinder. Drawings, of course, were made only with pencil. Again, graphic anticipation was found to be slightly in advance of actual seriation, while tactile seriation was found to be slightly behind visual seriation in its development. Piaget and Inhelder concluded that "anticipatory structures [as manifested in the drawings] grow out of the progressive

organization of actions, and that organization also structures perception, adapting it to its own needs" (Piaget & Inhelder, 1969, p.268).

Table II.6

Anticipation & Performance in Seriation: % of Subjects at Each Stage					
Age-->	4 yrs.	5 yrs.	6 yrs.	7 yrs.	8-9 yrs.
No. of subjects-->	19	33	19	10	7
Ia: Failure in anticipation	89	42	5	0	0
Ib: Global anticipation	11	55	73	20	0
Ic: Analytic anticipation	0	3	22	80	100
IIa: Failure in seriation	84	54	42	0	0
IIb: Success by trial-and-error	16	40	36	20	14
IIc: Operational seriation	0	6	22	80	86

#### The D. G. Phillips Group's Investigations Regarding Ordering

D. G. Phillips (1996, p. 147) reported five Ordering Task studies carried out by his research group. The percentage of subjects passing the task for each grade level is presented in Table II.7 below. (The age ranges of the students in the present study corresponded roughly to kindergarten, first grade, and second grade.)

Table II.7

Ordering by Length Task: % Passing				
Study	Grade	n	% Passing	Sig. Gender Differences
A	1	34	68%	no
	2	34	88%	
	3	34	100%	
B	2	36	39%	no
	4	36	69%	
	6	36	94%	
C	PreK	28	39%	no
	K	36	39%	
	2	36	86%	
D	K	342	26%	no
E	K	34	38%	yes
	1	36	92%	
	2	32	100%	

Study D used protocol, materials, and scoring categories identical to those used in the present study. In only one study were significant gender differences in performance found. In study E the number of females passing the task was significantly greater than the number of males who passed the task.

#### Other Pertinent Research Involving Ordering

In a fascinating case study, Leiser and Gillieron (1990), in an effort to further bridge the gap between the American cognitive science camp with its emphasis on procedures and the Genevan genetic epistemology camp which focuses more on

operational structures, unearthed data which were gathered from seven studies of length and weight seriation with children ranging from 5 to 12 years and one seriation study with adults all carried out by the authors between 1972 and 1976. They reclassified the responses according to type of procedure (algorithm or strategy) used to seriate the objects. They found as many as ten different ways of going about the seriation tasks which were used by subjects of various age levels. It is beyond the scope of this review to explain each procedure, but, suffice it to say that the authors concluded that, even though it may be possible to "teach" people the procedures for solving the seriation tasks, and even though the proponents of artificial intelligence have succeeded in programming computers to carry out some of the procedures, this does not mean that the subject or even the computer programmer "understand" the procedure. Understanding, they insisted, still requires that the person possess the operational structure.

Clark (1983) administered a seriation pre-test to a group of 44 kindergartners and 52 first graders ranging in age from 5.4 to 7.2 years who were living in Washington, D. C. The task materials consisted of ordering 6 sticks (compared to 12 straws or rods in the present study) without insertion. Sixteen percent of the kindergartners and 35% of the first graders passed the task. In the present study the percentage of kindergarten-age students who could at least order the rods, i.e., disregarding insertion, was 38% (as contrasted to 3% passing all parts) and the percentage of first-grade-age students was 68% (as contrasted to 10% passing all parts). (See Appendix G--those scoring 2 or above.) As part of Clark's post-test, a control group consisting of 20 kindergartners and first graders (a rather low N) were given a seriation task which required the insertion of only one rod (as compared to 3 rods in the present study). Ten percent of the students passed this task.

Kingma (1983b) called into question the Piagetian criteria for determining partial and operational seriators and he called for the development of "a new theoretical

framework about the acquisition process of seriation." In his study a seriation task was administered to 428 Dutch children ranging in age from 4.10 to 11.10 years. Subjects were asked to build a series with 10 tubes ranging in length from 10 cm to 14.5 cm by placing them upright on cylindrical pins. Of the children between 4.10 and 6.5 years, only 5 out of 103 (5%) were considered "nonseriators." Their solution strategies were analyzed. Many seriators used one or a combination of: direct comparison, perceptual comparison, trial-and-error, and, or verification of a transitive comparison (in  $A > B > C$ , the subject verifies perceptually that A is greater than C)--methods which Piaget attributed to partial seriators. Very few used concatenated transitive comparisons (after comparing A to B and B to C in  $A > B > C$ , a subject's actions show that he/ she knows that  $A > C$  without making a direct verification) which Piaget posited as a criteria for operational seriation.

Kingma's study, in this author's opinion, had two weaknesses. Firstly, although Kingma claimed to have followed Piaget's protocol very closely, his study did not adhere to the original task as described by Piaget (1965, Ch. VI). By having the cylinders placed vertically on a table, a baseline is automatically provided for the subject thereby greatly facilitating the task. Secondly, there was no insertion as described by Piaget. According to D. G. Phillips (1996), a simple series can be created with lower-order structures such as perceptual comparison (which is all that the task required of the Dutch children), but this is not operational seriation. It is insertion that forces the child to consider both  $>$  and  $<$  simultaneously. Also, the seriation task designed by Phillips and his colleagues goes beyond even insertion by requiring the coordination of two, corresponding but non-adjacent series and locating corresponding members of the seriated sets via counting.

In the above study of Dutch children, even though the seriation by length task was much simpler than the one used in the present study of Colombian children, Kingma (1984) did report a difference in performance amongst various age groups



(4.10 to 7.10 years) until second grade, after which all subjects from third through sixth grades (8.10 to 12.0 years) passed the task.

As another aspect of this same study, Kingma (1983a) examined the relationships amongst seriation, correspondence, and transitivity tasks. Kingma reported that some researchers considered these three operations to have a single, underlying concept which had led some schools in the Netherlands to use correspondence and transitivity tasks in place of seriation tasks in the development of preschool curricula and posttests. Kingma's study indicates that there are distinct concepts underlying each of the three tasks and that, therefore, correspondence and transitivity tasks should not be used as substitutes for the traditional seriation task based on Piaget's work. In the present study the ordering task required children to perform all three tasks: the child is first asked to seriate rods, then he/ she is asked to insert three rods into the series (transitivity), and finally to match a corresponding set of circles of various sizes and (after condensing the series of rods) to find the rods corresponding to selected circles by counting (D. R. Phillips, 1991, p. 140). Certainly this meets Kingma's standards for a comprehensive task for post-test purposes.

Baylor, Gascon, and Lemoyne (1973) and Baylor and Lemoyne (1975) in a series of experiments were able to isolate the factor causing the horizontal *decalage* between length and weight seriation (weight seriation is acquired about three years after seriation by length [Piaget, 1965]). In addition to the standard seriation tasks for length and weight a "hidden sticks" task was devised to simulate the weight task. Seriated sticks were placed in equal-length cigar tubes and the subjects had to order them by taking out only two sticks at a time from the tubes. (In the weight seriation task only two equal-sized boxes could be compared on a balance scale at once.) Results showed that, using the hidden sticks task, the *decalage* nearly disappeared, that the hidden sticks task was much harder than the standard seriation of length task. This confirmed the explanation given by the Geneva school for the three-year *decalage* between

length and weight seriation; namely, that the usual length seriation tasks are perceptually facilitated by the visual simultaneity of the perceived elements, whereas weight seriation, like the hidden sticks task, obliges the child to use more conceptual strategies because only two items can be compared at a time. The child is further constrained in the weight seriation task because all of the elements look and feel alike.

In comparison to traditional intelligence and achievement tests (Cattell form 1, Cattell form 2A and subtests from the PMA 5 to 7), Kingma and Koops (1983) found that, from a psychometric point of view, seriation, classification, and conservation tasks were superior predictors of number language, and equally good predictors of number line comprehension and verbal arithmetic. Both types of tests were found to be equally poor predictors of simple, mechanical forms of computation (addition, subtraction and reversal tasks). This latter result could have been influenced by the use of conservation of substance, length, and volume tasks instead of a conservation of number task. The latter has a closer correspondence to the development of number concepts (D. R. Phillips, 1991). The authors attributed the poor power of prediction to the mechanical nature of the computation tasks; a conclusion which is in agreement with the longitudinal study carried out by D. G. Phillips (1989) which indicated that superior ability with logical thinking tasks such as seriation, classification, and conservation did not result in significantly superior performance in the Math Computation section of the Iowa Test of Basic Skills (ITBS). However, they did predict superior achievement in the Math Problems and Math Concepts sections of the ITBS.

In relation to reading, DeYoung and Waller (1983) carried out a fascinating study which correlated the performance of kindergarten and first grade readers versus non-readers as determined by the administration of various subtests of the *Metropolitan Achievement Test-Primer* with their performance on a battery of Piagetian tasks in four areas: seriation (which included ordering by length), conservation, classification and perceptual decentering. They found that the scores on the Piagetian

tasks were significantly different for the readers and non-readers and they posited a possible "facilitative relationship" between Piagetian operative competence and learning to read. It is of interest to note that their findings were consistent with their earlier reviews of the literature (1976 and 1977) which tentatively yet consistently indicated a link between reading achievement at the primary grade level and performance on four categories of tasks involving concrete operational thought: seriation, conservation, classification, and perceptual decentering.

Contrary to the above study, Robinson's (1987) study of the relation between early readers' and pre-readers' performance on Piagetian tasks of seriation, conservation of number, conservation of quantity, multiple classification, and class inclusion, found no significant differences in three samples of kindergarten children between the performance of early readers and pre-readers on the selected concrete operational tasks. She concluded that "concrete operations" are neither necessary nor sufficient for beginning reading. According to Robinson, such contradictory results in studies attempting to find relations between concrete operations and reading are common. The equivocal results appear to confirm Ginsburg and Oppen's (1988, p. 248) conclusion that Piaget's theory had "little if anything to say about reading."

In response to Modgil and Modgil's 1976 finding that "the literature on Piagetian epistemological development among children from 6 to 11 years old is devoid of long-term observational studies in which each child is followed for more than 1 or 2 years," Oakes (1984) carried out just such a study. Forty-eight first graders were assessed on their performance on Piagetian tasks which included seriation, reversibility, classification, conservation of continuous quantity, and conservation of number. The tasks were administered again at the third grade and fifth grade levels in order to confirm or disconfirm "the idea that concrete Piagetian development occurs in individuals and can be assessed." Of the 48 children who were assessed in first grade 33 were also available for assessment in third and fifth grade. An examination of the

individual and group graphs of performance confirmed that "each child was able to perform at a higher age of task performance in the third grade than in the first and at a higher age of task performance in the fifth grade than in the third." It was also found that "marked individual differences exist in the patterns of concrete logical development among children." It was concluded that the findings "agreed with expected levels derived from Piaget's work and other verification studies" and "lend considerable support to Piaget's general theory about the course of epigenetic development" (p. 375).

Blevins-Knabe (1986) studied the development of 3- to 6-year-olds' ability to insert an item into a series. Learning tasks using arrays on a computer screen with positive and negative feedback were incorporated into the procedures. Seriation tasks used 8 sticks with a 1 cm difference in length between each. The baseline was implicitly given via the use of a small stand with a ledge for placing the sticks. Each child was asked to insert only one stick. No correspondence between two series was required. Several task variations were incorporated which simplify the task demands as compared to those of the standard seriation task just described. For example, one subtask used only four sticks. It was found that for most of the modified tasks the children solved them at about the same age regardless of task demands and regardless of the provision of extra experience. Blevins-Knabe concluded that the results support Piaget's and Halford's predictions of stage-related constraints on children's insertion skills, rather than Bullock's invariance hypothesis which claims that experience is a more important influence.

Blevins-Knabe's results are different from those of Koslowski (1980) who found the seriation of four sticks to occur at an earlier age than the seriation of ten sticks. The difference between the use of eight and ten sticks could possibly make a difference. The results are also different from D. G. Phillips' (1989) finding that increasing the amount of experience does enable concrete structures to be formed at an earlier age

while recognizing that there may be wide differences amongst individuals. The key difference between the studies was the *type* of experience provided. The Blevens-Knabe study provided a limited number of training sessions with sticks that were only two-dimensional and could not be manipulated with the hands. The series were pre-constructed for the child who was only given practice in insertion. The experience provided by D. G. Phillips with D. R. Phillips was extensive practice throughout the year with real sticks which had to be placed in a series by the child. In this author's opinion, Blevens-Knabe has unwarrantedly supported the notion of "age constraints" instead of "stage constraints."

Pasnak, et. al. (Malabonga, Pasnak, & Hendricks, 1994; Malabonga, Pasnak, Hendricks, Southard, & Lacy, 1995; Pasnak, R., 1987; Pasnak, Brown, Kurkjian, Mattran, Triana, & Yamamoto, 1986; Pasnak, Campbell, Perry, & McCormick, 1989; Pasnak, Holt, Campbell, & McCutcheon, 1991) have used seriation tasks along with classification and conservation tasks in their Piacceleration instruction studies; thus named because of the focus on accelerating the development of Piagetian constructs through learning-set training; especially with young children who are mentally retarded or who are cognitively lagging behind their peers. The instructional procedures included the use of a wide variety of manipulatives (one study used as many as 140 different materials), explanations and demonstrations of how to solve the problems, the liberal use of encouragement and tangible rewards, and persistent instruction during a relatively short time (usually 15 to 20 minutes, two or three days per week for three months) until a high level of mastery of the operations is attained. Before and after the treatment the students were evaluated using a variety of intelligence tests, school ability tests, academic achievement tests and concrete operations tests. Results showed that the mentally retarded children made significant gains in IQ, classification, and seriation. The cognitively-lagging children made significant gains in both seriation and classification scores and in academic achievement in the areas of mathematics

concepts and verbal comprehension. The authors attributed the success to the children being at a transitional age and stage in which they are poised for a spurt in cognitive development along with the variety of instructional techniques. What is not clear from the reports is which of the intervention factors was the most influential in causing gains. Was it the right vs. wrong feedback, the award system or the children's extensive actions on a wide variety of everyday concrete objects?

Two of the reports examined the issue of *near* and *far* generalization. The Malabonga, Pasnak, and Hendricks (1994) study found only *near* generalization in the sense that the children retained the ability to classify and seriate in tests that were *similar* to the instructional situation until the end of the school year. In the Malabonga, Pasnak, Hendricks, Southard, and Lacy (1995) study it was found that there was *near* and *far* generalization in relation to the children's ability to perform seriation and classification tasks in tests that were both *similar and dissimilar* to the instructional situation. There was also *far* generalization to the kind of reasoning assessed in intelligence tests.

In conclusion, like class inclusion, the mental structure for ordering is viewed in many different ways in the research: the signs of its onset, what factors constitute it, how it should be assessed, what factors influence performance, and so on. (For a summary of the critiques of the Piagetian Ordering Task see Section VI.)

### Research on Conservation of Number

#### Piaget's Investigations Regarding Conservation of Number

In *The Child's Conception of Number* (1965) Jean Piaget reported various studies concerning conservation of number with children from 3 to 9 years. Again, these appear to be exploratory. The number of subjects and statistical results were not

presented. The materials and protocols varied. The findings were generally the same for all of the experiments.

After investigating conservation of continuous substances using liquids, Piaget shifted to the study of conservation of discontinuous substances using beads. The containers that were used in the previous study were the same. Equivalence was established using containers of the same size and by having the subject drop in a bead for every one dropped in by the interviewer. Then the beads from one of the two containers were poured into one that was taller and narrower. Subjects were then asked which amount of beads would make the longer necklace. Non-conservers could not yet grasp the idea that a change in one dimension was compensated for by a change in another dimension. This lead them to believe that there was a change in number because "it's [the container is] narrow and they [the beads] go higher" (1965, p. 26). Conservers could coordinate two dimensions simultaneously: taller but thinner = same number (an inverse relation).

Two other types of experiments (which were similar to the present study) utilized sets of corresponding objects. In one approach the interviewer worked with the subject to establish one-to-one correspondence between the objects before transforming the configuration of one of the sets. The materials used in this approach had a functional relationship: bottles and glasses, vases and flowers, eggs and egg holders, and pennies and candies. In the second approach Piaget wanted to see how the subject spontaneously went about establishing correspondence before carrying out the transformation. The materials used for the two sets in this approach, instead of being different but functionally related, were the same: all counters, all buttons, or all matches. The configurations created by the interviewer which had to be reproduced by the subject were of various types: a set of objects placed randomly; parallel rows; shapes such as houses or circles composed of various amounts of counters; closed figures, such as a square or a cross, requiring a fixed number of counters; less familiar,

closed figures such as a rhombus; and single rows (the present study used this latter configuration). In general, the procedures consisted of placing two sets of objects in one-to-one correspondence with one another (in one task the objects were even counted out loud--which had no effect on the results). Then one of the sets was grouped close together or spread out. The subject was then asked which group contained more.

Subjects had less difficulty constructing figures which were based on perceptual numbers (1-5) and were familiar: triangle, square, etc. But, when the objects were placed randomly, or, when the objects were more numerous, beginning level subjects had difficulty duplicating the configuration. Excluding this exception, in general, all of the tasks showed students passing through three general stages: global evaluation, correspondence without lasting equivalence, and numerical correspondence with lasting equivalence.

In the first stage, children made global correspondences. Their experience did not compel them to decompose the whole. As long as the model figure and the subject's created figure looked similar or as long as a row was the same length, regardless of the density of the objects, the subject was satisfied. Even in the tasks in which the interviewer guided the subject to establish correspondence, as soon as one of the "look alike" figures was transformed, the subject lost the sense of equivalent amount. The perceptual cues of longer or denser overpowered the sense of correspondence and equivalence. The subjects at this level could not reverse, in their mind, the transformation in order to return mentally to the original correspondence. Piaget considered children's responses to be "amazing" in which, first, one set and then the other had "more," depending, for example, on the length of the row. He concluded that when a child at this stage says "six glasses" it does not have the same meaning as for an adult. He/ she may be able to apply the first six numerals to objects, but the truly numerical sense of six is not yet in place because the number of glasses



can change by changing the space that they occupy, i.e., spreading them out or grouping them close together.

At the second stage, subjects were better at making a more accurate copy of the model figure or row--even correct numbers of counters--but, when one of the sets was transformed, correspondence and equivalence of number vanished. The subject was still not able to link the new, apparently changed end-state with the former state of equivalence.

During the third stage, students easily established equivalence via one-to-one correspondence or counting and they could conserve number via compensation by observing that a row was longer but the objects were also more spread out, or vice versa, a row was shorter but closer together: "They're still the same. You've only put the bottles close together" (p. 47). In other words, they could coordinate two dimensions simultaneously: density and length (or, in the case of groups, width). Some subjects would also show conservation of number via reversibility, that is, they acquired the ability to imagine the transformation of the configuration going backwards to the way it was before: "Because before, those (his own) were in a bundle, and now you've put them like that (spread out), and these (the model) were spread out before, and now you've made them into a bundle" (p. 83). The number of objects in a set remains permanent for the conserving child, even when the configuration is changed, that is, the number of objects has now been differentiated from the space they occupy. Perception has now been dominated by reversible thought and the elements have become interchangeable units capable of being dealt with in a truly numerical sense.

#### The D. G. Phillips Group's Investigations Regarding Conservation of Number

D. G. Phillips (1996, p. 433) reported three Ordering Task studies carried out by his research group. The percentage of subjects passing the task for each grade level is

presented in Table II.8 below. (The age ranges of the students in the present study corresponded roughly to kindergarten, first grade, and second grade.)

Table II.8

Conservation of Number Task: % Passing				
Study	Grade	n	% Passing	Sig. Gender Differences
A	K	342	4%	no
B	1	113	14%	no
C	2	97	36%	no

These three studies used the same protocol, materials, and scoring categories as were used in the present study. No significant gender differences in performance were found.

#### Other Pertinent Research Involving Conservation of Number

Number conservation has been one of the most-studied problems in cognitive developmental psychology. In 1978 Murray surveyed more than 140 studies and hundreds of others were reviewed by Field in 1987 and by McEvoy and O'Moore in 1991 (Siegler, 1995). Because of the vast amount of literature on number conservation, this review will be necessarily limited to studies which this author considered to be of particular significance and, or relevance to the present study.

Siegler (1995) found general agreement in the research that 3- to 5-year-olds often perform more successfully under certain conditions: "if the rows include small rather than large numbers of objects; if the transformations involve addition or subtraction rather than neither adding nor subtracting; if the wording of questions is facilitative; if they think the transformation was accidental rather than intentional; or if

the children are trained through presentation of rules, feedback, modeling, social interaction with a conserving peer, discrimination learning sets, or any of a host of other instructional methods." He further noted that "several factors have been found to correlate with success in training procedures such as: age, prior relevant knowledge, and initial incongruity between gestures and verbal explanations" (pp. 227-228).

Siegel and Goldstein (1969) carried out a study to reconcile the conflicting results of Mehler and Bever (1967) and Piaget (1965) regarding the age at which children begin to show the ability to conserve number. Contrary to Piaget's finding that the age of the onset of conservation was 5-7 years, Mehler and Bever found what they considered to be conservation behavior in children as young as 2.4 years. Their procedure consisted of using M & M candies, which they considered to be more motivating than the beads used by Piaget. The candies were arranged in two rows: one consisting of six pellets 5 in. long and the other containing four pellets 8 in. long. The children were asked which row they preferred to eat. Of the 2.4- to 2.7-year-olds 100% answered correctly. The results were rejected by Piaget (Achenbach, 1969) because the procedure involved unequal rather than equal rows and, therefore, was not testing the conservation of a discontinuous amount. The procedure could also be criticized because of the use of small "perceptual" numbers and because the longer row was still within a small perceptual field of focus. Siegel and Goldstein, in their attempt to clarify the issues, requested verbal responses regarding *more*, *less*, or *same*. First they tested for understanding of these words and eliminated from the study the non-comprehending children. They also tested for recency--the tendency for young children to choose the last multiple-choice answer when faced with a complex situation. The material used was pennies placed in two rows of six each. The report stated that the transformation consisted of pushing one row close together but does not mention the relative lengths. Their results showed that 95% of 2.7-3.0 year-olds chose the last alternative response. This decreased with age. Not until ages 5-7 did a

sizable percentage of children show conservation of number and only 40% of the 5.7- to 6.1-year-olds passed the task. The authors concluded that their results supported Piaget's findings.

Rose and Blank (1974) devised a task for assessing conservation of number amongst subjects whose mean age was 6.3 years which included only one judgement regarding equality of number rather than two judgements; one before and one after the transformation. They proposed that "in the normal (non-experimental) course of events . . . one would never ask the identical question twice if a significant change had not occurred in the material that was being observed." In most Piagetian protocols, according to Rose and Blank, the subject "has just said that the rows are equal, then he is questioned again after witnessing the transformation. In these circumstances he may well feel that the second question itself seems to suggest that a new judgement is in order, with the result that he changes his answer" (pp. 499-500). In their study the investigator established equivalence between the two rows of objects and then, just before the transformation, said, "Look at the rows, watch what I'm doing." No verbal justification for the subject's answer was required. The "most striking result" that they found was that errors on conservation with the one judgement protocol dropped by 50% as compared to the two judgement task. They concluded that the social context in the two-judgement task cues the subject to interpret the request for a second judgement as a signal to change his/ her response, thereby obscuring the conservation ability of a large proportion of experimental populations.

The authors do not consider other factors which could have contributed to the increase in conservation ability. For example, the interviewer established equivalence rather than the subject. This casts doubt as to whether the child acknowledged "same amount," i.e., established equivalence, before the transformation. Also, because the subject was not asked for a verbal explanation of his/ her judgement, he/ she had a 50% chance of passing the task by guessing, or, he/ she could have used the empirical

strategy of counting to verify equivalence instead of a logical reason. Both of these factors could have increased the percentage of subjects passing the tasks.

In replication studies, the improved performance using the one-judgment format found in the study of Rose and Blank was replicated by Silverman (1979) and by Samuel and Bryant (1984) but was not replicated by Miller (1977) nor by Silverman and Briga (1981). Sophian (1995) proposed an alternative explanation for the social-interactive-cue phenomenon--that "children look for social-interactive cues mainly when they do not understand the problem well enough to work out a solution for themselves . . . they can only interpret the repetition of the comparison question on those problems as an indication that something important is different, and so they are inclined to change their response" (p. 576).

In the research of D. G. Phillips and his co-workers (1996), including the present study, the protocol for conservation of number did not include the repetition of the judgement question as described above by Rose and Blank. Rather, equivalence was established by the child. The interviewer, after placing 8 white blocks in a row, gave the subject 10 green blocks and then asked him/ her to "make a row of these blocks beside the row of white blocks so that there is one green block for each white block." Only once was the conservation question asked: "Are there more green blocks, more white blocks, or, are there the same number of green and white blocks?" In this way the social "cuing" context, i.e., the repetition of the same question, was avoided.

Irwin and Briga (1981), in order to test the claim of Gelman (1972) that children conserve as young as three years when using arrays containing 2-3 items, conducted four experiments with 3-year-olds in which variations of (1) the usual two-judgment protocol were used, (2) one element was covered, and (3) two elements were covered. Superior performance in the first two conditions was found. But performance dropped drastically to chance level in the third condition when two elements were covered. The investigators reasoned that the children were using an empirical strategy of obtaining

the numerosity of the arrays before and after the transformation, that in the second condition they were able to add 1 to the array to find the sum, but that in the third condition they could not add 2 to 1 because they had not yet developed commutative addition. They concluded that the logic required for conservation of number was not available to the 3-year-olds and that the results contradict those of Gelman.

In a duplication study with 4-5-year-olds, Starkey (1981) attempted to confirm or disconfirm Bryant's (1972 and 1974) model which claims that young children can conserve number by applying a rule which they use only when length cues are unavailable. In other words, children, according to Bryant, can conserve number when the perceptual miscue of transformed length does not interfere. The results confirmed that Bryant had failed to control for other irrelevant cues, especially the order-of-array-transformation cue which operates in non-equivalence conservation tasks when children judge as more numerous the array that was transformed first. And in Bryant's studies it was always the more numerous array that was first transformed, thus skewing the results in favor of responses which were misinterpreted as evidence of conservation of number. Starkey concluded that there is little evidence that young children can conserve number and strong evidence that children use a wider range of number-irrelevant cues or strategies than suspected by Piaget.

In Botswana, M. John, Dambe, Polhemus, and F. John (1983) studied the Piagetian task performance of 554 school children ranging in age from 6 to 14 years and living in urban, rural, and traditional village environments. The six tasks assessed ability in conservation of number, seriation, conservation of length, classification (which level is not apparent from the report), conservation of mass, and conservation of weight. In comparison to the results of the present study regarding conservation and seriation, the percentages of Botswanian subjects passing the tasks (shown in Table II.9) was much higher.

Table II.9

Botswana Study: Percentages Passing Tasks			
Age	n	Conservation of Number	Seriation
6	51	51%	16%
7	35	57%	31%
8	39	72%	44%

This difference in performance was possibly due to a difference in protocol. Unfortunately, the report did not include a description of procedures.

The investigators found that, in general, the Batswanian children use logical thinking strategies at different age levels and that the age at which conservation of number and seriation occur is about the same at which they take place in Euro-American cultures. In the other tasks the Botswanian children lagged behind their Euro-American counterparts.

Baroody and White (1983) studied the relation of counting skills and number conservation amongst 5- and 6-year-olds. They found that nearly all children who conserved number were also successful in a series of progressively more advanced counting skills culminating in the  $N + 1 > N$  principles. There was a small percentage of children, however, who were able to conserve number but were unable to appreciate the  $N + 1 > N$  rule. The authors concluded that counting experience may be an important vehicle for making explicit and extending intuitive notions of equivalence which begin as early as six months of age in relation to small "perceptual numbers." They pointed out, however, that counting experience may not be the only means of acquiring the conservation of number structure.

Halford and Boyle (1985) carried out five experiments using a unique methodology for assessing conservation of number, based on that of Bryant (1972).

Two parallel rows of 20 counters each were placed so that there was no one-to-one correspondence, yet the lengths of the rows were equal. One row was then transformed into one of four other arrangements with the same characteristics, i.e., one row, unlike the Piagetian task, was not lengthened or condensed into a group. Four different beginning displays were utilized with other more subtle variations being made from one experiment to another. The authors justified the change on the basis that the Piagetian task rightfully contains an ambiguous, "testing" transformation, but contains a biased post-transformation display which cues the subjects to assume that there was positive increase in quantity. Their procedure retained the ambiguous transformation while removing both pre- and post-transformation bias. The purpose of the study was to compare the performance of 3-4-year-old children with that of 6-7-year-olds in order to verify whether or not the younger children understand conservation of number. The results showed that the 3-4-year-old children base their judgements on cues contained in visual displays rather than on transformations. When there are no salient, visual cues they base their decision on information contained in the pre-transformation display. The 6- to 7-year-olds, on the other hand, did show a significant tendency to interpret the transformation itself (instead of just the display) and maintain their previous judgements, thereby showing some cognizance of invariance and the conservation of number. The authors concluded that, contrary to the claims of some researchers (Bryant, 1972; Gelman, 1972), 3-4-year-olds do not understand conservation of number. Their results support those theories that postulate age differences in conservation of number for children in the range of 3 to 7 years (Halford, 1982; Klair and Wallace, 1976; Piaget, 1965; Siegler, 1981; Siegler & Robinson, 1982).

Sophian (1995) conducted three experiments with 3-6-year-olds in order to examine the developmental relation between counting and number conservation. Results indicated that only the older 6-year-old children were able to conserve number



and that there was a close, though not necessarily causal relation between counting and number conservation. Both phenomena showed a protracted development from ages 3 to 6. The authors concluded that their results were compatible with a broader picture of children's developing numerical ability that is emerging from research reports, "a picture that agrees in some respects with both Piaget's account of a relatively late-emerging, logic-based, concept of number and the post-Piagetian view of early, counting-based, numerical competence" (p. 576).

Kaplan (1987) evaluated the performance of 58 kindergartners on a traditional number conservation task and a task requiring judgement of quantity without establishing initial equivalence. Forty-seven percent of the subjects passed the conservation task. For the other task the students were presented with two transparent plastic boxes, one with 8 checkers, the other with a variety of 8 plastic pieces. Each child was instructed to find out which container had more in it or whether the containers had the same amount. The experimenter apparently considered both tasks to be forms of number conservation referring to them as "traditional and nontraditional tasks of conservation of number." The performance on the two tasks was not closely associated. Twelve (39%) of the non-conservers succeeded in the task which did not first establish equivalence. Of the 27 partial or full conservers only 16 (59%) determined that the quantities in the transparent boxes were equivalent. Because of the lack of relationship between performances, the author postulated that the tasks are assessing different schemata. This apparently is the case. The "non-traditional" task appears to be evaluating the ability to establish equivalence rather than assessing conservation. This assumes that the student has at hand the ability to set up a one-to-one correspondence arrangement or can count and compare. In the present study the experimenter set up one row of 8 blue poker chips and the child was given a set of 10 white poker chips and was given the instruction, "I would like for you to make a row of these white circles beside the row of blue circles so that there is one white

circle for each blue circle." Much of the equivalence establishment procedure was set up for the children. They did not need to come up with it on their own. And even if they could, this would have nothing to do with whether or not the quantity of the set was conserved after a transformation involving the spreading out or the condensing of the configuration of the objects.

Tollefsrud-Anderson (1987) tested 148 preschoolers aged 4.0 to 6.0 years on a Piagetian number conservation task. She used a four-tier scoring system in which she found 39 "nonconservers," 23 subjects who made correct judgements but could give no explanation ("transitionals"), 4 children who could give an adequate explanation only after probing through questioning, and 51 subjects who gave correct judgements and adequate explanations ("conservers"). Half of the nonconservers and transitionals were given training which emphasized both what does and what does not change number with active participation of the students in counting exercises. Two weeks later a post-test was administered. Results showed that the students who received training had no advantage over those who did not. Both groups showed some improvement between the pre- and post-tests.

In a most fascinating, extensive, and elegant microgenetic study of number conservation, Siegler (1995) was able to isolate factors which enabled 5-year-olds to learn that focusing on transformations (spreading out or condensing one of two parallel rows of buttons, simply moving one of the rows backwards and then forwards to its original position, or adding or subtracting objects to one of the rows) instead of comparing the lengths of the rows or counting the objects in the two rows, was the more rapid and accurate way of explaining the answer "whether that row [the row closer to the subject presumably] then has the same or a different number of buttons." Ninety-seven children between the ages of 4.5 and 6.1 years were pre-tested using a number conservation task similar to the one used in the present study. Fifty-four percent of the children passed the task. The remaining 45 non-conserving

kindergartners were divided into three experimental groups and were given different types of intervention training: (1) feedback on correctness/ incorrectness only, i.e., no explanation of reasoning, (2) feedback plus request to explain own reasoning, (3) feedback plus request to explain experimenter's reasoning. One of several significant findings was that "being asked to explain the experimenter's reasoning produced considerably more learning than either of the other two procedures." One of the common characteristics of the children who did learn from this type of intervention was that they "generated multiple types of reasoning about conservation . . . both within and across trials." The authors hypothesized that giving multiple reasoning could be a sign of cognitive conflict which would indicate that these subjects were in a transitional phase. They also pointed out that the children who benefited most by explaining the experimenter's reasoning were probably drawing upon (1) their earlier experience with conserving number after transformation in sets containing small numbers of objects, (2) their ability to gradually and more frequently select the explanation which resulted in the greatest number of correct answers, and (3) their arithmetic ability in counting (to verify answers) and in addition/ subtraction which would tell them more readily that if none were added or subtracted that the amount would be the same. The authors concluded by considering how to take educational advantage of the positive results of "children's efforts to understand other people's reasoning,"<sup>15</sup> especially those children

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<sup>15</sup> In the Developmental Activities Program (D. G. Phillips, 1996; D. R. Phillips, 1991) right/ wrong feedback is not given and explanations are not presented in an effort to prevent the "stealing" of the structure which would reduce the student to mimicking a teacher's procedure to solve a certain type of problem in a certain type of situation with little generalization to other circumstances. But because students, during exploratory activities, are asked to explain their reasoning, the question becomes whether or not it would be possible to further mediate students' development of structures by (1) asking them to prove their answer--most often by returning the objects to their one-to-one-correspondence position or by counting, (2) suggesting these methods of proof if they are not automatically utilized, e.g., "Could you return the objects to their original places, or count them?", and then (3) going beyond these interventions by saying something like, "I knew the answer without returning the objects to their

in a transitional phase.

Bisance, Dunn, and Morrison (1995) examined the influence of school- vs. age-related variables on conservation of number and mental addition. They found that on number conservation tasks, performance improved as a function of age but not schooling. On mental arithmetic tasks, accuracy improved with schooling instead of age, but subjects' use of various solution procedures, such as retrieval or counting) was not influenced by schooling. The authors admitted that specific factors in the cultural milieu and particular instructional methods vary widely from society to society and from school to school and that they need to be better identified. The results of the three-year longitudinal study of D. G. Phillips (1989) indicated that methodologies used in schools can be changed in order to enhance the development of mental structures such as number conservation without sacrificing the learning of arithmetic skills. Unfortunately, "schooling" and "teaching" have come to be associated exclusively with explanation, demonstration, practice, drill, and finding the "one right answer"; practices which do not facilitate the formation of concrete operations.

In conclusion, this review shows how number conservation has been studied and defined in many different ways. There is a need for a common theoretical definition. This author agrees with Halford and Boyle (1985) that boundary conditions need to be set; that number conservation involves the ability to interpret and conserve an ambiguous transformation in which the subject judges quantity as a non-perceptual, intellectual system of beliefs, the absence of which forces the subject to use cues which are social or perceptual in nature. Hence, number conservation tasks should contain large rather than small, "perceptually quantifiable" sets. Such a definition would interpret the abilities of 3- to 4-year-olds, as worthy as they are, to correctly judge small quantities before and after transformations, as "pre-conservational"

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positions (or without counting). How do you think I knew that?" (This last question is from Siegler [1995, p. 239].)

because this same ability has not been demonstrated with large sets. (See Section VI for critiques of the Piagetian Conservation Tasks.)

### III. METHODS AND PROCEDURES

In this section a description will be given of the four tasks used to assess the four targeted mental structures--collections, class inclusion, ordering by length, and conservation of number. The pilot study will be reviewed and its results analyzed. The components of the principal study will then be delineated: the sample, the interview protocol and procedures, the scoring procedure and degree of reliability, the type of data collected for each subject, the use of the "clinical interview" as a unique assessment "instrument," and the type of statistical analysis chosen to examine the results.

The four tasks used in the present study were taken from *Structures of Thinking: Concrete Operations* by Darrell G. Phillips (1996): Task 1--Collections (for assessing pre-operational classificatory thinking), Task 2--Class Inclusion (for assessing Primary Addition of Classes [LG<sub>1</sub>]), Task 3--Ordering by Length (for assessing Addition of Asymmetrical Relations [LG<sub>5</sub>]), and Task 4--Conservation of Number (to partially test for the Additive Group of Whole Numbers [N<sub>1</sub>]).

The tasks were designed taking into account the following criteria:

- they are not solvable on the basis of perceptual or non-logical processes;
- significant amount of prior, academic, content knowledge is not required to give a correct response;
- subject responses are so elementary that the effects of verbal ability and motor skills are minimal;
- the interviewer ascertains throughout the task that the subject actually perceives and recalls necessary information;
- cuing behaviors are intentionally eliminated from the clinical interview (Kyhl, 1994).

A description of the structures tested by the tasks was given in Section I. A

description of the equipment, the procedures, the scoring categories, and the scoring sheets for each task is given in Appendices B-E. Two minor equipment changes were made. For the Class Inclusion Task, instead of 9 toy cows and 4 pigs; 9 toy crabs and 4 toy fish were used. The fish were identical and made of bright green plastic. The crabs were also identical and made of bright yellow plastic. These animals are very familiar to the children involved in the study who live in Barranquilla, a river port city which is also adjacent to the Caribbean Sea.

For the Ordering by Length Task, straws were used for the interviews carried out in the air-conditioned rooms at the Marymount School, but, in the fan-cooled room at the Fe y Alegría School, wooden, rectangular-prism-shaped dowels were used in order to prevent the materials from being blown away.

### Description of Tasks

The tasks and protocols used in the present study were more formal and more standardized in comparison with the flexible interview method used by Piaget and his colleagues. The intention was to obtain more consistency and reliability in the research. A brief, general description of each task follows. A more detailed description can be found in the four corresponding appendices.

#### The Collections Task

The Collections Task was administered by displaying on a table a set of plastic-coated, pastel-colored, tagboard shapes: small and large squares, small and large rectangles, small and large right triangles, isosceles triangles, and half-rings. After spreading out the mixed up shapes the initial instruction was, "Here I have some pieces of paper. Look at them carefully. I would like for you to put together the ones that

you think should go together." The child was then guided by questions, starting from the level of his/ her performance, to see how far advanced was his/ her pre-classificatory thinking. Graphic collections are characterized by side-by-side, non-overlapping "layouts" (no piles): small partial arrangements (G1), continuous alignments (G2), homogenous linear segments (G3), intermediate responses (G4), collective objects (G5), complex objects (G6a), and complex objects based on situational content (G6b). The types of graphic responses do not have a particular developmental order and for this reason they are reported as one category in the results. If the child began with a graphic collection, questions were then asked to invoke the demonstration of non-graphic collections. There is a possible transitional response called "mixture of graphic and non-graphic." This category is followed by four levels of non-graphic collections characterized by the use of overlapped, stacked piles which do occur in developmental sequence: NG1 in which the piles have different and overlapping criteria; NG2 in which there are numerous, small, but "clean" piles, i.e., they have no overlapping criteria; NG3 in which the child can make four clean piles based on shape or color but not both; and NG4 in which the child can shift the classification from color to shape or vice versa. Note that this is still not true classification because the subject does not have to be able to coordinate a superordinate and subordinate class simultaneously. He/ she only has to differentiate the attributes of the objects according to one attribute at a time: color or shape. Diagrams of the different types of arrangements made by children and more detailed descriptions of these response categories can be found in Appendix B.

### The Class Inclusion Task

In the task for primary addition of classes the child was first asked to identify the toy fish and crabs, the group of fish, the group of crabs, and the general group of



animals (fish and crabs together). Questions then tested for intensive exclusion: "Tell me, if we take all the crabs away will there be any animals left, or not?"; intensive quantification: "If we take all the animals away will there be any crabs left, or not?"; and extensive quantification: "Tell me, are there more crabs or more animals?"

In this task the children had to maintain the class of animals in their mind while subtracting the sub-class of crabs. Then they had to compare the sub-class of crabs, which existed in actuality before them, with the class of animals, which no longer existed in an undivided form because the crabs had been separated from the fish.

### The Ordering by Length Task

The Ordering by Length Task consisted of four parts. If a child successfully carried out one part the interview he/ she continued to the next. In Part A the child was asked to order a set of twelve dowels by length from the shortest to the longest. In Part B the subject was asked to insert up to three rods into the ordered set. In Part C a contextual story was created for a "lollipop" factory. The child was requested to match a corresponding set of round, size-seriated, pastel-colored circles made of tagboard with the seriated dowel rods. This required that the set of seriated dowel rods be broken up. In Part D the subject was asked to return the dowel rods to their original, side-by-side, seriated position. Then he/ she was asked to point to the circle which corresponds to the rod indicated by the interviewer. This was repeated with two other rods. Moving the rods was not allowed. Counting and maintenance of correspondence between the two sets were required to pass this last, and most difficult part of the test. This was the true test of the child's attainment of the complete structure which requires much more than simply putting a set of dowels in order.

## The Task for Conservation of Number

In this task for testing conservation of number the child was first presented with a row of 8 blue poker chips (circles) and then asked to take 10 white poker chips (circles) and "make a row of these circles beside the row of blue circles so that there is one white circle for each blue circle." After the equality of the two sets was established via one-to-one correspondence and interview questions, the extra white circles were set out of sight. The interviewer then proceeded to condense the blue circles into a group and the subject was asked whether there are more white circles, more blue circles, or if there were the same number of white and blue circles. After this the interviewer spread out the blue circles into a row approximately one meter long beside the shorter row of white circles. The same question was then repeated.

After each response to the interviewer's questions (in this task and at several points in the previous tasks) the child was asked to explain his/ her reasoning via questions such as, "Why do you think so?" It was the action of the subject on the materials plus the verbal expression of his/ her reasoning which indicated to the interviewer the sufficiency of the child's reasoning, the level of structure attainment, and the final decision regarding the categorization of the response.

### Pilot Study

The purposes of the pilot study were to determine (1) whether the interviews should be conducted in English or Spanish, (2) if the interviewer was properly carrying out the interview and the scoring, (3) whether the length of the interview was appropriate for the age of the subjects, (4) whether the length of the interview would allow for the administration of a fifth task for the conservation of length, and (5) whether

the use of fish and crabs for the Class Inclusion Task would be appropriate.

The selected tasks were field-tested with 19 students (6 boys and 13 girls): 6 students between 5.6 and 6.5 years, 3 students between 6.6 and 7.5 years, and 10 students between 7.6 and 8.6 years of age. All of the children attended Marymount School in Barranquilla, Colombia. Students were selected randomly.

The four Piagetian-type tasks administered were:

Task 1: Collections

Task 2: Class Inclusion

Task 3: Ordering by Length

Task 4: Conservation of Number

Data from the pilot study are presented in Tables III.1-III.4 below.

Table III.1

Pilot Study--Collections Task Data--% in Each Response Category							
Ages	N	G1-G6	GN	NG1	NG2	NG3	NG4
5.6-6.5	6	67	17	0	17	0	0
6.6-7.5	3	100	0	0	0	0	0
7.6-8.6	10	20	20	0	10	30	20

Table III.2

Pilot Study--Class Inclusion Task Data: Percentage in Each Response Category (0-8)										
Ages	N	0	1	2	3	4	5	6	7	8
5.6-6.5	6	0	33	17	17	0	33	0	0	0
6.6-7.5	3	0	0	0	0	0	100	0	0	0
7.6-8.6	10	0	0	0	0	0	50	0	0	50

Table III.3

Pilot Study--Ordering by Length Task Data: Percentage in Each Response Category (0-8)										
Ages	N	0	1	2	3	4	5	6	7	8
5.6-6.5	6	0	33	0	33	17	17	0	0	0
6.6-7.5	3	0	33	0	33	0	0	0	33	0
7.6-8.6	10	0	0	0	20	0	60	10	0	10

Table III.4

Pilot Study—Conservation of Number Task Data: Percentage in Each Response Category (0-5)							
Ages	N	0	1	2	3	4	5
5.6-6.5	6	0	33	50	0	0	17
6.6-7.5	3	0	0	33	0	33	33
7.6-8.6	10	0	0	60	10	0	30

The results of the pilot study indicated that the children felt more comfortable in their native Spanish language rather than English which is their language of instruction.

In order to determine the proficiency of the interviewer the following materials were sent to Dr. Darrell G. Phillips for his evaluation: a videotape of five sample interviews conducted in English and covering all four tasks along with the corresponding task sheets, the task sheets for the other fourteen subjects, a data summary sheet, and particular questions regarding student responses.

The feedback was positive and included suggestions such as "pursuit questions" to follow through on questionable student responses. It was also pointed out that the tasks needed to be administered in random order.

Regarding the length of the interviews, the duration ranged from about 15 to 30

minutes depending on the responses of the child. It was decided that, considering the time available from classes in school and the attention span of the children, administering four tasks was possible and comfortable, but five tasks would be too many.

Concerning the substitution of fish and crabs for cows and pigs, the children had no trouble identifying the fish and crabs.

### Principal Research Study

#### The Sample

For the present study, during the months of May and June of 1996, a sample of 120 subjects was selected from the population of students in the Marymount School and the Fe y Alegría School--Barrio Las Malvinas both of which are located in Barranquilla, a major Caribbean sea-river port on the north coast of Colombia, South America which has a population of over one million inhabitants. The two schools were chosen for their accessibility. Marymount is a bilingual, private, Catholic-oriented school whose student population is composed of Colombian, urban, upper-middle and upper socioeconomic class children of business people and professionals. The Fe y Alegría School is a charity project operated by a Catholic, non-governmental organization and is located in Barrio Las Malvinas, a lower socioeconomic class neighborhood situated in the poverty belt surrounding the southern part of the city. The student population is composed of Colombian children of laborers, factory workers, and domestic workers. (See Appendix F for human subjects authorization documents.)

Originally, the sample of subjects was to have only been taken from the Marymount School. But, at the end of the school year, it was decided that those students who had not yet accomplished the minimum academic objectives for the year

should have two weeks of reteaching. The more accomplished students would stay at home during the two weeks. Because it would no longer be possible to maintain a random sample, the quality of the study was endangered. Also, it would have been disadvantageous for students to lose valuable reteaching time when pulled out of class for the interview. Therefore, it was decided that the sampling would be completed by interviewing students at another school.

The Director of the Fe y Alegría School offered to help. The school would be open well into June. With the assistance of the staff it became possible to reach the upper end of the proposed sample range of 90-120 subjects. Unfortunately, the need to change populations occurred after the requisite number of 5.6-6.5 year-olds had been interviewed at the Marymount School. Hence, the sub-samples from each school are not equivalent in number at each age range. However, even though the social classes of the sub-samples were quite different and the numbers uneven, the objectives of the study were still accomplished: to verify, in a population of Colombian students, whether or not there would be differences amongst children of the same age range, differences across age ranges, and differences between boys and girls.

Within each school the subjects were selected for the study using a stratified sampling method (Gay, 1996) in order to assure an approximately even number of boys and girls. Names on class lists were coded as being male or female. Students were selected randomly from each category, alternating between boys and girls. In addition to providing a balanced male-female sample, this method also assured that neither boys nor girls were given a time-of-day-related advantage. Although particular students were volunteered by the teachers and themselves, they were never accepted. Random selection was always used except for a few students who were not tested because they were absent or overly nervous.

The students' ages ranged from 5 years, 6 months to 8 years, 6 months. They were enrolled in grades kindergarten, first, and second. The composition of the

research samples is described in Tables II.1.5-III.7 below.

Table III.5

Composition of the Total Sample Used in the Principal Research Study				
GENDER	AGE			
	5.6-6.5	6.6-7.5	7.6-8.5	Total
Females	19	23	17	59
Males	21	17	23	61
Total	40	40	40	120

Table III.6

Composition of the Marymount School Sample Used in the Principal Research Study				
GENDER	AGE			
	5.6-6.5	6.6-7.5	7.6-8.5	Total
Females	19	13	5	37
Males	21	12	9	42
Total	40	25	14	79

Table III.7

Composition of the Fe y Alegría School Sample Used in the Principal Research Study				
GENDER	AGE			
	5.6-6.5	6.6-7.5	7.6-8.5	Total
Females	0	10	12	22
Males	0	5	14	19
Total	0	15	26	41

## Interviews

All students were interviewed in Spanish, their native language, in a private room. The rooms used at the Marymount School were air-conditioned. The room at the Fe y Alegría School was fan-cooled. (In order to enhance attention in tropical settings, provision for some kind of cooling is necessary.) At Marymount School the interviews were carried out during the early morning. At the Fe y Alegría School, even though the interviews took place in the afternoon, the children were attentive because, there being two sessions--morning and afternoon--with separate student bodies and faculties, they had just started their school day when the interviews began.

The four tasks were administered in a random order to the children. On special scoring sheets (See Appendices B-E.), the interviewer recorded relevant student responses in the form of drawings of arrangements made with objects or transcriptions of answers to questions. Also, all verbal interaction portions of the interviews were recorded on audiotape. Task protocols were closely followed in order to assure that all subjects received the same instructions and questions.

## Scoring Procedure and Scoring Reliability

This author scored the tasks after each interview and double-checked the scoring at a later date. Inter-rater agreement was determined from a random sample of 20 interviews checked by a Spanish-speaking person trained in the use of the four tasks which were used in the present study. A comparison between the author's and the second rater's scoring revealed 95% agreement on Task 1 (collections) and 100% inter-rater agreement on tasks 2, 3, and 4 (class inclusion, ordering, and conservation of number, respectively).



## Instrumentation

The data recorded for each subject consisted of: age (year and month), academic grade level, gender, responses, and scores for the four Piagetian-type tasks each of which tested the formation of a particular mental structure: collections, class inclusion, ordering, and conservation of number. These structures lay the foundation for the child's understanding of number.

The clinical interview is considered to be a superior data-gathering instrument than those which rely only on pencil and paper. The manipulation of objects gives the interviewer insights into what a child is thinking far beyond what can be expressed in words or written numerals. The verbal responses give insights beyond what can be demonstrated with action on objects. Additionally, any vague, uncertain, or even overly certain responses can be pursued by making additional statements or asking further questions. The structured protocols and scoring criteria give consistency and allow for statistical analysis while the option for pursuing certain student responses provides flexibility. To a certain degree the interviewer becomes the data-collecting instrument (Maykut and Morehouse, 1995). The justification and advantage is that the object of investigation--the human subjects and their mental reasoning--is the most complex entity thus far known to exist in the universe; hence, the most appropriate instrument is another, equally complex entity--a human interviewer who is sensitive to subtle signs which indicate whether or not a structure has been formed by a child, or, at what stage the structure is in its development. The disadvantage, in addition to the time required for lengthy individual interviews, is that the "human instrument," in order to reduce error, must undergo sophisticated training in order to thoroughly understand the mental structures and then he/ she must practice to the point of precision. (Both Piaget and D. G. Phillips [1996] recommend a year of daily practice.) Thus, the clinical interview generates both quantitative and qualitative information. Although it is a much

more challenging instrument to use than a written test involving only the counting of correct and incorrect responses, via the multiple avenues of data gathering that it provides--(1) the manipulation of objects, (2) the protocol questions and verbal responses, (3) the use of pursuit statements and questions, and (4) the understanding of structures that the interviewer brings to the task--it proves to be a very sophisticated and adequate instrument. (See Appendices B-E for a detailed description of the protocols, equipment, scoring criteria, and scoring sheets, and see Section I for an explanation of the four structures investigated in the present study.)

## Statistical Analyses

### Research Question I

Is there a difference in task performance amongst subjects of the same age range?

There was no relevant statistical test to determine whether or not there was a significant difference in task performance amongst subjects of the same age range (5.6-6.5, 6.6-7.5, 7.6-8.6 years). No group or sub-group was being compared with another. Therefore, there was no theoretical expectancy frequency. For research of this type which is non-parametric in nature, a bell curve distribution may not be assumed. Nor may it be assumed that equal numbers of subjects would be expected to score in the same category (Siegel & Castellan, 1988). Therefore, the answer to this research question was based on a comparative analysis of the results, i.e., the number of subjects scoring in each category.

### Research Question II

Is there a difference in task performance amongst the three different age ranges?

The Chi-Square Test for  $k$  independent samples was used to determine whether

or not there was a statistically significant difference in task performance amongst the three different age ranges (Siegel & Castellan, 1988).

### Research Question III

Is there a difference in task performance between males and females?

Whether or not there was a statistically significant difference in the performance between males and females was computed using  $X^2$  for two independent samples (Siegel & Castellan, 1988).

## IV. RESULTS

In this section the data will be summarized in various types of tables: summary tables for all four tasks--collections, class inclusion, ordering by length, and conservation of number; individual tables for each separate task; and tables which present gender-related data. The statistical analysis of each set of data will be presented and then discussed in the light of each of the three research questions: Is there a difference in task performance amongst subjects of the same age range? Is there a difference in task performance amongst the three different age ranges? Is there a difference in task performance between males and females?

The interview responses were scored according to the pre-set scoring criteria presented in Appendices B-E. The analytical and statistical methods indicated in the previous section were then used to examine the results.

The total number and percentage of subjects passing each task, i.e., scoring in the highest category, and the number not passing each task, i.e., scoring in any category below the highest, is shown in Table IV.1.

The number and percentage of subjects passing each task by age range is presented in Table IV.2.

For each of the four tasks and for each age range, data regarding the number of subjects and percentage attaining the various levels ranging from no understanding, to partial understanding, to passing the task (the last column to the right) are presented in Tables IV.3-6.

The results of task performance by gender are shown in Tables IV.7-11.

The following items of raw data are summarized in Appendix G: subject's number, school, age, gender, and score on each of the four tasks.

### Data Summary Tables

The total number and percentage of subjects passing each task, i.e., scoring in the highest category (NG4 for collections, 8 for class inclusion, 8 for ordering, and 5 for conservation of number), and the number not passing each task, i.e., scoring in any category below the highest, is shown in the following table.

Table IV.1

Number and % of Subjects Passing and Not-Passing Each Task					
	Passing		Failing		
Task	n	%	n	%	Total
Collections	0	0	120	100	120
Class Inc.	9	8	111	92	120
Ordering	6	5	114	95	120
Cons. No.	13	11	107	89	120
Total	28	6	452	94	480

The following table presents the number and percentage of subjects passing each task by age range.

Table IV.2

Number and Percentage of Subjects Passing Each Task by Age Range							
		5.6-6.5		6.6-7.5		7.6-8.5	
Task	N	n	%	n	%	n	%
Collections	40	0	0	0	0	0	0
Class Inc.	40	2	5	4	10	3	8
Ordering	40	1	3	4	10	1	3
Conserv. No.	40	5	13	6	15	2	5
Total	160	8	5	14	9	6	4

### Data Tables for Each Task

The four tables that follow present, for each of the four tasks and for each age range, data regarding the number of subjects and percentage attaining the various levels ranging from no understanding, to partial understanding, to passing the task (the last column to the right).

Table IV.3

Collections Task Data: Total Number and % in Each Response Category													
		G1-G6 <sup>16</sup>		GN		NG1		NG2		NG3		NG4	
Age	N	n	%	n	%	n	%	n	%	n	%	n	%
5.6-6.5	40	9	23%	3	8	0	0	21	53%	7	18%	0	0
6.6-7.5	40	12	30%	2	5	0	0	19	48%	7	18%	0	0
7.6-8.6	40	9	23%	2	5	0	0	26	65%	3	8	0	0

Table IV.4

Class Inclusion Task Data: Total Number and % in Each Response Category										
Category→		0	1	2	3	4	5	6	7	8
Age	N	n-%	n-%	n-%	n-%	n-%	n-%	n-%	n-%	n-%
5.6-6.5	40	0=	5=	2=	4=	0=	27=	0=	0=	2=
		0%	13%	5	10%	0%	68%	0%	0%	5%
6.6-7.5	40	1=	1=	4=	1=	0=	29=	0=	0=	4=
		3%	3%	10%	3%	0%	73%	0%	0%	10%
7.6-8.6	40	0=	6=	4=	1=	0=	26=	0=	0=	3=
		0%	15%	10%	3%	0%	65%	0%	0%	8%

<sup>16</sup> Because there is no developmental sequence for the different graphic responses, data for G1-G6 are reported together as one category.

Table IV.5

Ordering Task Data: Total Number and % in Each Response Category										
Category-->		0	1	2	3	4	5	6	7	8
Age	N	n-%	n-%	n-%	n-%	n-%	n-%	n-%	n-%	n-%
5.6-6.5	40	2= 5%	23= 58%	2= 5%	6= 15%	1= 3%	5= 13%	0= 0%	0= 0%	1= 3%
6.6-7.5	40	2= 5%	11= 28%	0= 0%	3= 8%	4= 10%	14= 35%	2= 5%	0= 0%	4= 10%
7.6-8.6	40	3=8%	13= 33%	0= 0%	2= 5%	9= 23%	9= 23%	0= 0%	3= 8%	1= 3%

Table IV.6

Conservation of Number Task Data: Total Number and % in Each Response Category													
Category-->		0		1		2		3		4		5	
Age	N	n	%	n	%	n	%	n	%	n	%	n	%
5.6-6.5	40	6	15%	12	30	14	35	2	5	1	3	5	13
6.6-7.5	40	1	3	9	23	18	45	3	8	3	8	6	15
7.6-8.6	40	4	10%	15	38	17	43	1	3	1	3	2	5

### Gender-Related Data Tables

The following five tables show the results of task performance by gender. The first table presents the results for all four tasks on a pass/ non-pass basis. The last four tables give a more thorough breakdown of performance for each task separately based on the full range of scoring categories.

Table IV.7

Number and Percentage of Subjects Passing Each Task by Gender									
Subject Information		Tasks							
		1: Collect.		2: Cl. Inc.		3: Ordering		4: Cons.No.	
Gender	N	n	%	n	%	n	%	n	%
Female	59	0	0	3	5	3	5	6	10
Male	61	0	0	6	10	3	5	7	11
Total	120	0	-	9	-	6	-	13	-

Because so few subjects passed each task, the richness of the data, i.e., the scores indicating a wide range of performances, is lost by considering only the number of responses in the highest categories. Therefore, the following tables (IV.8-IV.11) are presented in order to show the full range of responses according to the various scoring categories.

Table IV.8

Collections Task: Number and % of Subjects Scoring in Each Category by Gender													
Subject Information		Categories											
		G1-G6		GN		NG1		NG2		NG3		NG4	
Gender	N	n	%	n	%	n	%	n	%	n	%	n	%
Female	59	14	24%	4	7	0	0	29	49	12	20	0	0
Male	61	16	26%	3	5	0	0	37	61	5	8	0	0
Total	120	30	25%	7	6	0	0	66	55	17	14	0	0



Table IV.9

Class Inclusion Task:																
Number and % of Subjects Scoring in Each Category by Gender																
Subject Information		Categories														
		0		1		2		3		4	5		6	7	8	
Gender	N	n	%	n	%	n	%	n	%	n	n	%	n	n	n	%
Female	59	0	0	4	7	8	14	3	5	0	41	69	0	0	3	5
Male	61	1	2	8	13	2	3	3	3	0	41	67	0	0	6	10
Total	120	1	1	12	10	10	8	6	5	0	82	68	0	0	9	8

Table IV.10

Ordering Task:																			
Number and % of Subjects Scoring in Each Category by Gender																			
Subj. Info.		Categories																	
		0		1		2		3		4		5		6		7		8	
	N	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%
F	59	4	7	22	37	1	2	4	7	10	17	12	20	1	2	2	3	3	5
M	61	4	7	24	39	1	2	7	11	4	7	16	26	1	2	1	2	3	5
T	120	8	7	46	38	2	2	11	9	14	12	28	23	2	2	3	3	6	5

Table IV.11

Conservation of Number Task:													
Number and % of Subjects Scoring in Each Category by Gender													
Subject Information		Categories											
		0		1		2		3		4		5	
Gender	N	n	%	n	%	n	%	n	%	n	%	n	%
Female	59	3	5	24	41	24	41	2	3	0	0	6	10
Male	61	8	13	12	20	25	41	4	7	5	8	7	11
Total	120	11	9	36	30	49	41	6	5	5	4	13	11

### Research Question I

Is there a difference in task performance amongst subjects of the same age range?

To determine whether or not there was a difference in task performance amongst subjects of the same age range, scores were categorized and comparatively analyzed. On a pass/ non-pass basis, Table IV.2 indicates that on the Collections Task there was no difference of performance because no student performed the task at an NG4 level (spontaneously grouping the tagboard pieces into four piles by color and then shifting to shape, or vice versa). On the other three tasks, the percentage of students of each age range who passed the tasks ranged from as low as 3% to no higher than 15%. Hence, the vast majority of students at each age range also performed the same--they did not pass the tasks. However, by using this approach to analyzing the data, as stated previously, much of its richness is lost due to over simplification.

Another way to view the data is to examine the breakdown of performance by categories of response as shown in Tables IV.3-IV.6. An analysis of the results presented in these tables indicates that subjects scored in a wide range of categories; that there were: (1) children who had barely begun to form the structures, (2) children who were at various levels of partial structure formation, and (3) a small percentage of children who had completely formed the structures.

### Research Question II

Is there a difference in task performance amongst different age ranges?

To determine whether or not there was a difference in task performance amongst the three different age ranges, a chi-square test for  $k$  independent samples (Gay, 1996, p. 503) was applied. The three age ranges were placed along the vertical axis of

each chi-square table (rows). Scoring categories were placed on the horizontal axis (columns). In order to meet the requirement that "when  $r$  is larger than 2 (and thus  $df > 1$ ), the  $X^2$  test may be used if fewer than [no more than] 20 percent of the cells have an expected frequency of less than 5 and if no cell has an expected frequency of less than 1" (Siegel & Castellan, 1988, p. 123), it was necessary to collapse some of the adjacent, related scoring categories.

A chi-square test was performed for each of the four tasks and are presented separately in Tables IV.12-IV.15. Table IV.16 summarizes the four separate chi-square analyses. To determine whether or not the differences between observed and expected frequencies in performance amongst the three age ranges are significant, the chi square value is compared with the chi square critical value. If the former is greater than the latter then there is a significant difference between observed and expected proportions of responses amongst the age ranges (Gay, 1996).

Table IV.12

$X^2$ for Differences in Performance among Age Ranges for Task I—Collections				
Category--> Age Range	GI-GN	NG1-NG2	NG3-NG4	Total
5.6-6.5	12	21	7	40
6.6-7.5	14	19	7	40
7.6-8.6	11	26	3	40
Total	37	66	17	120

$X^2(4 \text{ df}) = 3.432$      $\alpha = .05$      $X^2 \text{ critical} = 9.48$

Table IV.13

$\chi^2$ for Differences in Performance among Age Ranges for Task 2--Class Inclusion			
Category--> Age Range	0-4	5-8	Total
5.6-6.5	11	29	40
6.6-7.5	7	33	40
7.6-8.6	11	29	40
Total	29	91	120

 $\chi^2(1 \text{ df}) = 1.453$  $\alpha = .05$  $\chi^2_{\text{critical}} = 3.841$ 

Table IV.14

$\chi^2$ for Differences in Performance among Age Ranges for Task 3--Ordering				
Category --> Age Range	0-1	2-3	4-8	Total
5.6-6.5	25	9	6	40
6.6-7.5	13	7	20	40
7.6-8.6	16	2	22	40
Total	54	18	48	120

 $\chi^2(4 \text{ df}) = 18.166$  $\alpha = .05$  $\chi^2_{\text{critical}} = 9.488$

Table IV.15

X <sup>2</sup> for Differences in Performance among Age Ranges for Task 4--Conservation of Number				
Category--> Age Range	0-1	2	3-5	Total
5.6-6.5	18	14	8	40
6.6-7.5	10	18	12	40
7.6-8.6	19	17	4	40
Total	47	49	24	120

X<sup>2</sup>(4 df) = 7.632 $\alpha = .05$ X<sup>2</sup> critical = 9.488

Table IV.16

X <sup>2</sup> Values Calculated for Four Tasks by Age Range			
Task	X <sup>2</sup>	X <sup>2</sup> Critical	Sig. Difference
Collections	3.432	9.488	no
Class Inclusion	1.453	3.841	no
Ordering	18.166	9.488	yes
Conserv. of No.	7.632	9.488	no

The results indicate that there was a statistically significant difference in the performance amongst the three age ranges only on the Ordering-by-Length Task (addition of asymmetrical relations).

### Research Question III

Is there a difference in task performance between males and females?

To determine whether or not there was a difference in task performance between females and males, a chi-square test for  $k$  independent samples (Gay,

1996, p. 503) was applied. Gender was placed along the vertical axis of each chi-square table. Scoring categories were placed on the horizontal axis. Because the number of rows was not greater than two, the requirement that "when  $r$  is larger than 2 (and thus  $df > 1$ ), the  $X^2$  test may be used if fewer than [no more than] 20 percent of the cells have an expected frequency of less than 5 and if no cell has an expected frequency of less than 1" (Siegel & Castellan, 1988, p. 123), it was not necessary to collapse some of the adjacent, related scoring categories.

A chi-square test was performed for each of the four tasks which are presented separately in Tables IV.17-IV.20. Table IV.21 presents an alternative calculation of the  $X^2$  statistic for the Conservation of Number Task. Table IV.22 summarizes the five separate chi-square analyses. To determine whether or not the differences between observed and expected frequencies in performance between females and males is significant, the chi square value is compared with the chi square critical value. If the former is greater than the latter, then there is a significant difference between observed and expected proportions of responses between females and males (Gay, 1996).

Table IV.17

$X^2$ for Differences in Performance between Females and Males Task 1: Collections													
Subject Information		Categories											
		G1-G6		GN		NG1		NG2		NG3		NG4	
Gender	N	n	%	n	%	n	%	n	%	n	%	n	%
Female	59	14	24%	4	7	0	0	29	49	12	20	0	0
Male	61	16	26%	3	5	0	0	37	61	5	8	0	0
Total	120	30	25%	7	6	0	0	66	55	17	14	0	0

$X^2(5 df) = 4.051$

$\alpha = .05$

$X^2$  critical = 11.070

Table IV.18

X <sup>2</sup> for Differences in Performance between Females and Males																
Task: Class Inclusion																
Subject Information		Categories														
		0		1		2		3		4	5		6	7	8	
Gender	N	n	%	n	%	n	%	n	%	n	n	%	n	n	n	%
Female	59	0	0	4	7	8	14	3	5	0	41	69	0	0	3	5
Male	61	1	2	8	13	2	3	3	3	0	41	67	0	0	6	10
Total	120	1	1	12	10	10	8	6	5	0	82	68	0	0	9	8

X<sup>2</sup>(8 df) = 6.947 $\alpha = .05$ X<sup>2</sup> critical = 15.507

Table IV.19

X <sup>2</sup> for Differences in Performance between Females and Males																			
Task 3: Ordering																			
Subj. Info.		Categories																	
		0		1		2		3		4		5		6		7		8	
	N	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%	n	%
F	59	4	7	22	37	1	2	4	7	10	17	12	20	1	2	2	3	3	5
M	61	4	7	24	39	1	2	7	11	4	7	16	26	1	2	1	2	3	5
T	120	8	7	46	38	2	2	11	9	14	12	28	23	2	2	3	3	6	5

X<sup>2</sup>(8 df) = 4.302 $\alpha = .05$ X<sup>2</sup> critical = 15.507

Table IV.20

X <sup>2</sup> for Differences in Performance between Females and Males Task 4: Conservation of Number													
Subject Information		Categories											
		0		1		2		3		4		5	
Gender	N	n	%	n	%	n	%	n	%	n	%	n	%
Female	59	3	5	24	41	24	41	2	3	0	0	6	10
Male	61	8	13	12	20	25	41	4	7	5	8	7	11
Total	120	11	9	36	30	49	41	6	5	5	4	13	11

$$X^2(5 \text{ df}) = 12.150 \quad \alpha = .05 \quad X^2 \text{ critical} = 11.070$$

The results of the chi square test indicate that there was no significant difference in performance between males and females on tasks 1, 2, and 3: Collections, Class Inclusion, and Ordering. However, for the Conservation of Number Task, there was a statistically significant difference in performance between boys and girls with the boys outperforming the girls. However, the statistical difference was not great. A closer examination of the Table IV.20 above shows that 8 more boys than girls scored in the highest three categories (3-5) and that 6 more girls than boys scored in the lowest three categories (0-2). In a sample of 120 subjects these differences cannot be considered that great. Although the data do not require the application of Siegel's principle that no fewer than 20 percent of the cells should have an expected frequency of less than 5 and no cell should have an expected frequency of less than 1, when it is applied, the results shift to a difference that is not statistically different (Siegel & Castellan, 1988). In Table IV.21 categories 3 and 4 were collapsed because their cells all had expected frequencies of less than 5 and these constituted 33% of all cells.



Table IV.21

X <sup>2</sup> for Differences in Performance between Females and Males Task 4: Conservation of Number--Cell 3-4 Collapsed											
Subject Information		Categories									
		0		1		2		3-4		5	
Gender	N	n	%	n	%	n	%	n	%	n	%
Female	59	3	5	24	41	24	41	2	3	6	10
Male	61	8	13	12	20	25	41	9	15	7	11
Total	120	11	9	36	30	49	41	11	5	13	11

X<sup>2</sup>(4 df) = 8.142    oc = .05    X<sup>2</sup> critical = 9.488

Regardless of the degree of statistical difference, the fact that there was a greater difference in performance between boys and girls on the Conservation of Number Task than on the other three tasks, is worthy of note.

Table IV.22 presents a summary of the X<sup>2</sup> values for all four tasks by gender including the two alternative ways of determining X<sup>2</sup> for the Conservation of Number Task.

Table IV.22

X <sup>2</sup> Values Calculated for Four Tasks by Gender			
Task	X <sup>2</sup>	X <sup>2</sup> Critical	Sig. Difference
Collections	4.051	11.070	no
Class Inclusion	6.947	15.507	no
Ordering	4.302	15.507	no
Conserv. of No. (no collapsing)	12.150	11.070	yes
Conserv. of No. (3 & 4 collapsed)	8.142	9.488	no



## V. DISCUSSION

This section will present a discussion of the performance of same-age subjects on each of the four tasks in comparison to their North American peers, the relation between the percentage of subjects performing a task and the age range, the differences in performance as related to social-class, the differences in performance in relation to gender, the educational implications and limitations of the present study, and recommendations for further research.

### Task Performance by Subjects of the Same Age Range

Tables IV.3-14 show the wide range of performance by students of the same age level. However, inspection of figures IV.1 and IV.2 shows that very few subjects at any of the three age ranges passed the tasks. Although the purpose of the present study was not cross-cultural in nature, because the performance of the sample was so low, it is pertinent to compare the data of the present study with similar studies carried out in the United States (D. G. Phillips, 1996, pp. 45, 79, 147, & 433) as presented in the Table V.I below.

For the Collections Task the percentages were not that different. For the Class Inclusion Task only the second grade's percentage of passing subjects seems to be very low compared to U.S. groups of similar age range. For the Ordering Task all of the age groups' scores were comparatively low. For the Conservation of Number Task the Colombian kindergarten group's percentage was more than three times greater than the percentage of passing subjects in the U.S. study. The first grade percentages were similar, but the second grade percentage of the Colombian sample was far lower than that of the U.S. study.

Table V.I

Research Data for Four Structure from Similar Studies								
Ages <sup>17</sup> & Tasks	Present Study		A: Other Study		B: Other Study		C: Other Study	
Collections	n	%	n	%	n	%	n	%
5.6-6.5	40	0	342	3				
6.6-7.5	40	0	113	2				
7.6-8.6	40	0						
Class Inc.								
5.6-6.5	40	5	56	25	34	12		
6.6-7.5	40	10	34	53	36	28		
7.6-8.6	40	8	36	69	97	33	32	50
Ordering								
5.6-6.5	40	3	36	39	342	26	34	38
6.6-7.5	40	10	34	68	36	92		
7.6-8.6	40	3	34	88	36	39	36	39
Conserv. No.								
5.6-6.5	40	13	342	4				
6.6-7.5	40	15	113	14				
7.6-8.6	40	5	97	36				

Also, an examination of the data from the other studies shows very different percentages amongst studies of same-aged groups. These differences can be attributed to changes in task protocol and materials (D. G. Phillips, 1996) or to differences in the children's experiences which they brought to the interview situation; for example, different backgrounds in working with manipulatives, in explaining their

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<sup>17</sup> These three age ranges are approximately equivalent to kindergarten, first grade, and second grade in the United States.

thinking, or in being interviewed in a one-to-one situation by an adult (Siegler, 1995).

#### Relation Between Percentage of Subjects Performing a Task and Age Range

Inspection of Table IV.28 reveals that in only 25% (3 of 12) of the samples and sub-samples' performance on the four tasks was there any significant difference of performance amongst the age ranges. On the one hand, one might expect that more students would perform better on the tasks with each passing year. On the other hand, it must be kept in mind that the structures do not develop simply with maturation over time, that is, they are not innate. If appropriate opportunities to act on objects are not available to the subjects, it cannot be assumed that the structure will develop spontaneously. As was discussed in relation to Table 1.1 (Epstein, 1979), even some high school students, college students, and adults have not yet formed all of the concrete structures.

As Piaget (1967) stated:

The clearest result of our research on the psychology of intelligence is that even the structures most necessary to the adult mind, such as the logico-mathematical structures, are not innate in the child; they are built up little by little. . . . There are no innate structures: every structure presupposes a construction. All these constructions originate from prior structures. (pp. 149-150)

#### Social-Class-Related Differences in Performance

Another factor affecting the results could be the greater percentage of lower-socioeconomic status (SES) children in the older sub-groups of the sample: 0% in 5.6-6.5-years group, 38% in the 6.6-7.5-years group, and 65% in the 7.6-8.6 years

group. The percentage of students passing any of the tasks did not increase significantly with age. In fact, the lowest performance was that of the oldest group: 5.6-6.5 years--5%, 6.6-7.5 years--9%, 7.6-8.6—4%. In the youngest sub-group there were no lower-socioeconomic-class children. But in the oldest two groups there was a mixture of socioeconomic levels which permits the comparison of the number of students who passed at least one task as expressed in the following table.

Table V.2

Percentage of Subjects from Upper and Lower Socioeconomic Levels Passing at Least One Tasks			
	N (6.6-8.6 yr.)	n Passing	% Passing
Lower SES Level	41	4	10
Upper SES Level	39	16	41

Compared to lower-SES-level subjects, four times as many upper-SES-level subjects passed at least one task.

Other investigators have also found this difference in performance between socioeconomic levels. In their review of the literature on various intellectual differences between lower and upper SES children, Stodolsky and Lesser (1967) and Deutsch (1973) found that the lower class children performed consistently lower than did the higher class children. Yando, Seitz, and Zigler (1979) found that difference in intellectual and personality characteristics were more strongly associated with differences of social class rather than ethnic group membership. Hall and Kaye (1980) found social class differences on several measures of learning and intelligence for both White and Black 6 to 9-year-olds. Using Piagetian measures of reasoning, seriation, classification, causation, and conservation, to test 233 Black children between the ages of 6.9 and 12.9 years, Bardouille-Crema, Norcross Black, and Feldhusen (1986) found that higher SES children outperformed lower SES children on all tasks.

However, Ginsburg and Russell (1981) found no difference related to social class amongst preschool and kindergarten children who tested for mathematical thinking.

Another factor related to socioeconomic conditions is the familiarity of the children with adult-child interviews. Mwamwenda and Mwamwenda (1989) point out that children in some African cultures may perform more poorly than Western children because they are unaccustomed to the features of the Piagetian interview: asking for an explanation for an answer the interviewer already knows, being challenged, etc. The Marymount subjects were accustomed to being tested individually by an adult and being asked for explanations. It could have been that the Fe y Alegría group, because they do not have the human resources and budget for regular, one-on-one assessment, were less familiar with the interview situation. However, this author found the subjects to be quite verbal, although some were shy, and the few children who did show tension or nervousness were not included in the study.

Complicating the influence of socioeconomic level is the issue of exposure to one versus two languages. The upper SES subjects were exposed to both Spanish and English. The lower SES subjects were exposed to only their native Spanish language. Saito-Horgan and Hayes (1994), in a study of six-year-old Japanese and American children's performance on four Piagetian tasks--collections, class inclusion, conservation of mass, and conservation of area--found that "all subjects (Japanese and American) exposed to two languages performed significantly better . . . as compared to subjects exposed to one language." They found no significant differences between "(1) Japanese with one language/ one culture and Anglo-Americans with one language, and (2) Japanese with two languages/ two cultures and Anglo-Americans with two languages." However, in a later study, Saito-Horgan (1995) found that a group of bilingual-trained, lower SES, Hispanic children (ages 6-10) showed no significant advantage over a similar, monolingual group on a set of Piagetian tasks--collections, class inclusion, conservation of mass, and conservation of area.

## Gender-Related Differences in Performance

In the present study males performed slightly better than females on the Conservation of Number Task. On the other three tasks there were no significant differences in performance between males and females.

D. G. Phillips (1996) reported whether or not there were gender-related differences in studies which used protocols similar to those in the present study. For the Collections Task, in neither of the two studies reported were there gender-related differences. Eight studies were reported for the Class Inclusion Task, none of which showed significant differences between male and female performance on tasks. Of the five studies reported for the Ordering by Length Task, only one showed a significantly greater number of females passing than males. For the Conservation of Number Task, none of the three reported studies showed a gender-related difference.

Other Piagetian-based studies have reported no statistically significant, sex-related differences in performance on tasks (Dodwell, 1962; Lovell & Ogalvie, 1960). Smedslund (1964) and Dettrick (1974), who studied class inclusion in particular, also reported no gender-related differences in performance.

However, other investigators of Piagetian-type tasks have reported sex-related differences (Elkind, 1961; Goldschmidt, 1967). Shayer and Wylam (1978) found that males' performance was significantly superior to that of females' on tasks of spatial relationships, volume and density. Donoso (1983) found that a significantly greater number of males passed the task for Location of a Point in Three Dimensions. Of the 46 concrete operations studied by D. G. Phillips and his colleagues, only one--Rectilinear Order--a projective spatial task (PR0<sub>5</sub>), was found to consistently show males outperforming females (D. G. Phillips, 1996).

The differences that have been found have been attributed to both socialization and heredity. Socially, boys and girls are often encouraged to engage in



different types of games and activities. Genetically/ biologically it has been found that the male and female brains form differently from infancy which could result in different patterns of intellectual functioning (Kimura, 1992).

In studies which are not Piagetian-related, it has been found that women out-perform men in these areas: perceptual speed, ideational and verbal fluency, fine-motor coordination, and mathematical calculations. Men were found to outperform women in other areas: spatial tasks (which is consistent with the finding of D. G. Phillips mentioned above), target-directed motor skills, disembedding figure from ground, and mathematical reasoning (Kimura, 1992).

### Educational Implications

In relation to differences in task performance amongst students at the same age level, the wide range of performances found in the present study implies that an individualized curriculum and pedagogy are needed for the development of logical thinking. Even if students engage in the same activities, the teacher's expectations cannot be the same. The child who cannot group and regroup a collection of objects based on shape and then color, cannot be expected to mentally generate the idea that a person can be placed in one category such as "student" and then proceed to spontaneously place this same person in other categories such as "child of parents," "brother of a sister," "citizen of the country," or "member of human race." In general, he/ she cannot be expected to understand that there are many ways to organize (classify) the same group of objects, events, ideas, or people.

Regarding individual differences in class inclusion ability, if a child cannot compare two hierarchical levels of a classificatory system such as crabs and animals, then he/ she cannot be expected, for example, to compare and contrast the relations existing amongst the concepts of neighborhood, residential area, city, and urban area;

nor such hierarchically organized concepts as ringed worms, invertebrates, animals, living things, created things.<sup>18</sup> In a class with a wide range of performance on class inclusion tasks, it could also be assumed that there would be different performances on math tasks which depend on an understanding of the superordinate-subordinate organization of the number system such as place value, the base 10 system, systems with other bases, the decimal system, money, and operations involving carrying and borrowing (D. G. Phillips, 1991).

Concerning differences of performance in ordering or seriating by length, one implication is that the teacher can expect wide differences in children's understanding of related aspects of number: that counting is not just the recitation of a string of words; that each number is simultaneously greater than the one before it and lesser than the one after it; and that each number has a unique place in the series of all numbers. As was mentioned in the discussion of Piaget's model of development, Piaget stressed that a child's concept of number depends on the coordination of this concept of ordinality with that of cardinality.

Cardinality, which refers to the number of objects that a number refers to, cannot be considered firmly in place if the number of objects in a set is understood by the child to have changed simply because the configuration of the objects changes. The idea of a number needs to be conserved in spite of other transformations. Five, for example, must conserve its "fiveness" whether or not we are talking about five big elephants, five little mice, or a variety of five zoo animals. It should remain five whether or not these animals are strung way out in a line or bunched together. Hence, conservation of number is vital to understanding mathematics. Having students without this structure would even place constraints on what didactic materials could be profitably used. For example, when using base 10 blocks, a common set of manipulatives used in first and

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<sup>18</sup> I have chosen these examples from my own frustrating experience trying to teach these required curricular concepts to 8-year-olds over a three-year period.

second grade, a child may not understand that a 10 rod has the same value as a set of 10 unit cubes which are spread out. He or she may or may not assume that the latter set is greater in number depending on whether or not this structure has been formed in his or her mind. According to D. R. Phillips (1991):

Conservation of number is the cornerstone of the additive group of whole numbers. The child who is not yet conserving number (does not know that a set of objects has the same number no matter what the arrangement and who cannot justify this deduction with a logical reason) *should not be working on addition and subtraction with paper and pencil*. This child hasn't even internalized basic relationships and consistencies among objects; what possible sense can symbols standing for these objects make? (p. 259)

The data showing little positive change in task performance from one age range to the next has an important implication for education; namely that, if the formation of logical structures is considered to be important, then they cannot be left to be developed by chance. Granted, even in classrooms with a traditional approach, many children develop the structures on their own (Wadsworth, 1984). But, as has also been stated previously, it cannot be assumed that everyone will develop them. Therefore, in order to promote and track the mental development of all children, a special program is needed with special materials, a special time, and specially-trained teachers. That such a program does help students to develop structures sooner than students in traditional classrooms was confirmed by a three-year longitudinal study directed by D. G. Phillips (1989) which compared students in the Developmental Activities Program (a Piagetian-based program designed to facilitate the children's construction of mental structures) with students in more traditional classrooms which emphasized the use of textbooks and worksheets in science and math instruction.

In general, an important implication is that teachers need to be knowledgeable about the range of developmental levels amongst the children in their classroom and to ascertain whether or not the curriculum is requiring performance which depends on children having a particular structure. David Elkind comments:

Most of those who shape curricula, lack a developmental approach, . . . They keep floundering around, looking for new approaches and new theories to give them relevance. They do not appreciate that knowing child development is the best way to develop curriculum materials. (as cited in Alben, 1980, pp. 4-5)

Where necessary, the curriculum will need to be adjusted so that it follows more closely the natural pattern of development of logical thought. And if objectives and activities cannot be changed for more appropriate ones or more individualized ones, then, at least, the teachers' expectations can be more individualized and more realistic.

Another, related, yet more subtle implication for education is the use of school time. Some educators believe that Piaget's view of children's cognitive development is too negative (Crain, 1985); that young children are portrayed as being intellectually inept; and that greater attention needs to be given to what children "can" do well. Hence, they may decide to engage young children in problem-solving activities which can be solved by means such as rote counting, skip counting, counting backwards and forwards, etc., which do not require the use of logical structures. To teach such problem-solving techniques is worthwhile and practical. Indeed, in the present study the interview sheets for the Conservation of Number Task indicate that the vast majority of the students who answered correctly obtained the answer empirically via counting. When asked whether or not the number of poker chips had changed after the transformations, they simply counted and then answered, "No." For a

"correct-answer-oriented" educational system, this approach would be fine because "it solved the problem" and produced the "right answer." However, the children did not use logic. They did not use logical necessity by observing that "no chips had been added or taken away so the amount must be the same (so I don't even need to count)." They did not use reversibility by stating something like, "They were the same before and you just spread them out, so they must be the same now." Nor did they use compensation by saying, for example, "It looks like there are more in the longer line, but, they are still the same because there is just more space between the chips."

In other words, for many of the children in the present study thinking was limited to counting. The question then arises, "How much time during the school day should be devoted to solving problems via counting or other pre-logical means?" Should it be so extensive that no time is allowed for children to act on objects of their own choice for as many sessions as they choose? The implication, then, is that time is needed for both; time for children to enjoy solving problems via rudimentary means and to develop their counting ability (in the Conservation Task it was noted that several children miscounted the chips); and time to develop the mental structures which will help them to use more sophisticated, logical solutions in the future.

As regards gender, this research and many others indicate that both boys and girls can develop mental structures. Perhaps they develop at different rates, but the potentiality is the same. This finding is especially important for those persons who, and those cultures which, are sexually biased. The educational implication is that both boys and girls can benefit intellectually by engaging in the same cognitive curriculum and using the same materials and activities.

The present study is one of a continuing body of Piagetian-based research which continues to place pressure on the educational enterprise to accommodate itself to the developmental differences found amongst children. Dr. Barry J. Wadsworth, professor of psychology and education at Mount Holyoke College and author of *Piaget for the*

*Classroom Teacher and Piaget's Theory of Cognitive Development* sums up the challenge:

It is clear that the sixty years of research conducted by Jean Piaget on the development of knowledge in children has had a profound impact on psychological and educational theory. That we know more about how children develop mentally, how they learn, how they think, and how they reason because of Piaget's work is unquestionable. What is questionable is whether the rich knowledge Piaget left us will ever trickle down to the classroom teacher in a significant way and whether what trickles down will be distorted. . . .

Indeed, to submit all children to the same curriculum with the same expectations is to invite, if not guarantee, failure and can be viewed as educational neglect. (1984, p. 219)

#### Limitations of the Study

One of the possible limitations was the validity of the tasks, i.e., whether or not the task was really testing the targeted structures. D. G. Phillips (1996) noted that in some of his studies, he and his fellow researchers thought they were examining one structure and later found that actually a different structure had been tested. He also identified basic structures which are still in need of a valid task for their assessment. However, for the tasks used in the present study, there is considerable consensus that they do indeed assess the targeted structure (Kyhl, 1994).

Another limitation was the testing instrument, that is, the "human instrument" or interviewer--the author of the present study. In spite of the initial training that this author/ interviewer received, the additional practice in which he engaged, the positive results of the pilot study, and the high inter-scorer reliability, there is no doubt that the

present study would have benefited by having had an interviewer with at least a year of daily interview practice as recommended by Piaget and D. G. Phillips. Also, the fact that this author was both the principal investigator and the only interviewer could have produced biased results (Gay, 1996). As a veteran teacher who has always delighted in seeing kids "get the right answer," and as a researcher struggling to describe subject responses "as they are" rather than "as previous studies would predict them to be," it is possible that the interviewer accepted too many correct answers at face value and failed to ask more pursuit questions in order to ascertain the strength of a correct stance of the subject.

Regarding the sample, because subjects were drawn from two, disparate, social classes<sup>19</sup>, the experience that each group brought to the tasks was undoubtedly different. The higher class children probably had richer physical environments with which to interact; that is, more manipulative toys, kits, objects, collections, etc.; and they were probably more consciously oriented to educational purposes.

Intersubject communication of task details was also an unknown factor. During the study, to have been chosen for the interview became a source of prestige amongst the children and there was much "chit-chat" about what went on during the testing sessions. This might have resulted in some children passing on to others what *they* thought the "right answer" was supposed to be. If a student responded as told by a fellow student rather than following his/ her own understanding, the results would have been affected. However, as a teacher with years of experience working with children, this author did not get the feeling that this was happening. Besides, very few children passed any of the tasks which greatly reduced the likelihood that "right answers" were being communicated.

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<sup>19</sup> This, along with the fact that the interviews were conducted in the children's native language using familiar materials, could be considered a strength of the study because it provided a more representative sample of Colombian children.

Still other factors which could have limited the study, but which are nearly impossible to control for when working with human subjects, include: the subject's state of health, the degree of motivation to "do one's best," time-of-day preferences for mental activities, and affective factors generating hope- vs. fear-related feelings in relation to the interview situation. Regarding the latter, with shy children who feel overpowered by the authority and presence of an unfamiliar adult, they might respond to an interviewer who is trying to pursue a sufficient reason with a challenging statement like, "It sure looks like this row has more. Look how long it is!" by thinking, "He/ she is an adult so he/ she must be right" and then saying, "I think there are more." Of course, many other children have no problem with asserting that they believe something that is contrary to an adult's observation.

As a final note on possible limitations of the study, Piaget (1965) himself admitted the multiple constraints imposed by the content of the task:

We noticed wide differences in the results of the various tests of cardinal correspondence, showing that we never succeed in measuring understanding of this correspondence in its pure state and that the understanding is always with respect to a given problem and given material, (p. 149)

He further specifies just some of the intervening factors: "the words used, the length of the instructions given, their more or less concrete character, the relationship between the instructions and the individual experience of the child, the number of elements involved, the intervention of numbers the child knows, etc., etc." (1965, p. 149).

### Recommendations for Further Research

In relation to "local knowledge" about the populations involved in the present



study, research on the four structures—collections, class inclusion, ordering, and conservation of number--needs to be extended to older age groups in order to find out if and when the majority of students are able to master the tasks. This data could then form part of a local data bank which could be used as base-line data for future intervention programs or as guidelines for age-level expectations for the "local culture."

In the realm of "general knowledge," cross-cultural studies are needed which try to isolate those factors which contribute to one culture's superior performance over another's on particular tasks.

In relation to the development of the particular mental structures investigated in the present study, research is needed to determine the correlation between students' level of performance on the developmental tasks and their ability to organize and manipulate objects and information in the school curriculum. For example, in relation to the collections structure the question arises: Are students who are able to spontaneously group and regroup a collection of geometric figures of various colors by shape and then color, better able to spontaneously group and regroup sets of minerals, plants, and animals according to various criteria such as color, size, shape, kind, function, etc.?

Regarding the class inclusion structure, further investigation is needed to answer questions such as these: Do students who have demonstrated class inclusion ability on the assessment task have a deeper understanding of social studies concepts such as the subordinate and superordinate relations amongst concepts such as nation vs. region, urban vs. rural areas, city vs. suburb, and industrial vs. residential vs. business areas? Do they also have a better understanding of the hierarchical relationships involved in the base-10 system, place value, number systems with other bases, the decimal system, money, and operations involving carrying and borrowing?

In relation to the ordering structure, studies are needed which search for the correspondence between success on the task which tests this structure and students'

understanding of the ordinal aspect of number. Likewise, correlative studies are needed to determine the correspondence between success on the task which assesses conservation of number and children's understanding of the cardinal aspect of number and their understanding of the relationship between the mathematical symbol system and physical objects.

In the area of curriculum, more research is needed on the relationship between specific logical operations and academic content in areas such as science, social studies, and mathematics. The question, "Just what would a developmentally appropriate curriculum look like in these subjects?" needs to be answered and then such a curriculum, once implemented, would need to be tested and compared to more traditional approaches in which all children of approximately the same age range, regardless of developmental level, carry out the same laboratory, textbook and worksheet assignments at the same time and with the same expectations of mastery.

In the area of pedagogy, investigations are needed which pinpoint the most beneficial teacher-student, verbal and body-language interactions for the development of mental structures: Which questions are the most fruitful? When is intervention or non-intervention preferable? What attitudes and behaviors are especially beneficial? and so on. Also, comparative studies are needed regarding the selection of didactic materials for the development of mental structures: Which materials are the most advantageous for the formation of a particular structure for a particular type of learner? Which materials lend themselves to the simultaneous development of a variety of structures?

Correlative and causal studies are needed between the fields of mental structure development and learning styles—preferences for learning during different times of day; for working alone or with others; for analytical vs. global thinking; for tactile, kinesthetic, visual, or auditory pathways for processing experience, etc.

Many other related areas of inquiry could be proposed because the field is so

vast and there are so many questions begging for an explanation. Without a doubt, humankind's endeavor to refine the human mind is still in its infancy.

SECTION VI:  
SUPPLEMENTARY ESSAY  
CRITIQUE AND ASSESSMENT OF THE PIAGETIAN PARADIGM

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## VI. CRITIQUES AND ASSESSMENT OF THE PIAGETIAN PARADIGM

### Introduction

The purpose of this section is to summarize the critiques and the assessments of Piaget's work in order to further contextualize the present investigation within the field of cognitive developmental psychology. It will necessarily be limited to general issues and to those particular concerns which are related to the mental structures examined in the present study. It is beyond the scope of this summary to set forth all of the points and counterpoints for each issue. However, some commentary is given when considered appropriate.<sup>20</sup> Topics have been ordered so that, as much as possible, they will flow naturally from one to another in a general direction of broad to specific issues. In general, researchers do not deny the reality or the reliability of the developmental phenomena which Piaget and his followers reported. Nor do they deny their importance. What investigators do have trouble with are how the phenomena were studied and the epistemological and theoretical interpretations that were given to them by Piaget (Carey, 1987; Driver, 1982; Flavell, 1963; Gelman and Billargeon, 1983). Brodzinsky, Sigel, and Golinkoff (1981) identified four types of scholars who have dealt with the critical issues: "(1) those working close to Piaget's original formulations, (2) those who stay within the framework but see the need for revision, (3) those who see flaws in the theory and attempt to challenge its basic assumptions, and (4) those who reject the theory *in toto*." Brodzinsky, Sigel, and Golinkoff consider themselves and even Piaget himself to fit into the second category because they consider no theory to be sacrosanct. The authors stated:

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<sup>20</sup> Readers interested in a full seale debate are referred to *Jean Piaget: Consensus and Controversy* (Modgil & Modgil, 1982).

Since psychological theories are products of their era and since change is intrinsic to all living forms, change in the theory is inevitable. Change comes not only from those working within the theory, but even from the originator himself. Piaget as a scientist has become his own change agent because "the sciences by their use of methods of experimentation and deduction solve some problems and constantly give rise to new ones . . . [Piaget, 1971, p. 232]" . . .

No theoretical system is complete or necessarily eternally valid. Scientific inquiry by its very nature is an open system. For our purpose, the critical issue is the way in which scientific inquiry has influenced Piaget's theory. After some early efforts at replicating Piaget's ideas, his theoretical formulations became increasingly pervasive among developmental psychologists. Although the theory is still pervasive, a number of current attempts are being made to revise the theory, in light of a body of empirical disconfirmations which have appeared in the last decade. (p. 4)

### Epistemology

Brodzinsky, Sigel, and Golinkoff (1981) consider the primary epistemological question of developmental psychologists to be, "How do we come to know?" The answer to this question determines the definition of knowledge within the field and influences how knowledge acquisition will be investigated. There are two cosmological perspectives which have had a great impact on developmental psychology--the organismic model and the mechanistic model. The mechanistic model views behavior as being the effect of antecedent causes external to the organism. The organismic model, while accounting for efficient causation, the source of which is located in the organism's external environment, emphasizes final causation oriented toward subjective aim resulting in a certain degree of self-determination of the organism,



depending on its ontological level, as it engages in the process of its becoming (Jordan & Shephard, 1972; Whitehead, 1978). The mechanistic model of human development leads to a *naïve realist* epistemology which denies the existence and causative power of subjective mental states, while the organismic model provides the foundation for a *constructivist* epistemology. Piagetian theory operates within the organismic/developmental paradigm (Brodzinsky, Sigel, & Golinkoff, 1981; McFarland & Grant, 1982).

The Piagetian constructivist theory views the human being as a self-organizing, self-regulating, self-actualizing system. The individual organizes, structures, and restructures experience throughout life based on the schemes of thought available to him/ her at the moment. These structures are modified and enriched as the person interacts with the physical and social world. Knowledge is viewed as being constrained by the available mental schemes or structures which give a particular quality to a person's mental world. Structures are never static. They are dynamic in nature; continually being transformed qualitatively and increased quantitatively through the constructivistic functions of assimilation, accommodation, and equilibration. Hence, for Piaget, knowledge is never an exact copy of reality. It is a derived entity--constructed via structures through the active interchange between the person and reality. Thus, the main contribution of Piaget's genetic epistemology is the discovery that knowledge is obtained and maintained through the ongoing process of mental construction (Brodzinsky, Sigel, & Golinkoff, 1981; McFarland & Grant, 1982).

This Piagetian constructivist epistemology is opposed by mechanistic behaviorism with its logical positivistic epistemology in which developmental levels are viewed as the accumulation of new knowledge which is acquired in continuous, bit-by-bit, incremental fashion. Change is considered to be gradual with no sudden emergences of new behavior. Change is analyzed on an antecedent-consequent basis (Brodzinsky, Sigel, & Golinkoff, 1981).

Because Piagetian theory relies on non-observable structures or mechanisms of the mind, naturalists such as D. C. Phillips would rather have Piaget concentrate on observable behavior and its products. Piagetians, however, posit that their theoretical structures are powerful constructs for enhancing understanding of behavior and for generating hypotheses capable of directing empirical research. They argue that the physical sciences also rely on the positing of non-physical structures to explain phenomena, e.g., center of gravity and force field theory (Noddings, 1995, p. 108).

Another form of the traditional behavioristic model is the position that knowledge construction can be explained by reciprocal behaviorism (Bandura, 1977). In this epistemological theory, a person's construction of knowledge is controlled simultaneously by internal, cognitive rules and by stimulation from the external environment (Brodzinsky, Sigel, & Golinkoff, 1981).

Piaget's form of constructivism is also diametrically opposed by the radical constructivist position which rejects the existence of an independent, pre-existing world outside of the mind of the individual, cognizing subject who organizes his/ her experiential world via an internal, adaptive, constructive process (Noddings, 1995).

Within the constructivist school itself there are other critics of Piaget's epistemology. Social constructivism criticizes Piaget for not giving due attention to the child's construction of knowledge and intellectual structures via interaction with the social environment. Even though Piaget acknowledged the influence of social dialogue, he never articulated the social mechanisms underlying intellectual development. Social constructivists have investigated the circumstances in which social interaction can induce conflict or disequilibrium that can stimulate the accommodation of an underdeveloped mental structure (Forman, 1993).

Still other educators have chosen to ignore constructivism as the building of mental "constructs" or "structures." They have preferred to focus on the construction of

"knowledge" through hands-on, manipulative, everyday experiences (Forman, 1993).<sup>21</sup>

The epistemological views of Lev Vygotsky's socially-mediated learning also contrast to those of Piaget. Vygotsky posited that cognitive development cannot be understood apart from the cultural and human influences which interact with the mind during its development. His theory, unlike Piaget's, is not based on principles of endogenous construction or equilibration. He viewed knowledge and intelligence as moving from outside the person to the inside whereas Piaget argues that they move from the inside out. Hence, for Piaget, interior thought stimulates expression through language, while, for Vygotsky, language influences thought. A "word" for Vygotsky is a microcosm of human consciousness which guides and forms part of a concept. These basic epistemological divergences generate several other contrasting theoretical and practical differences the discussion of which is beyond the scope of this summary (Armstrong & Wilson, 1993; Forman, 1993; Richardson & Sheldon, 1988, p. viii).

Another aspect of Piaget's genetic epistemology which has been criticized is his attempt to show a parallel relationship between the ontogenesis of scientific knowledge in childhood and the history of science. Carey (1987) argued that "insofar as there are developmental constraints on the kinds of theories children can construct, the parallels between the characterization of conceptual change in childhood and in the history of science are limited" (p. 160). From another angle, in order for scientific theories of the past to be constrained in their development, the minds of the ancient theorists would need to be constrained developmentally. They would have to think like children throughout their adult lives. Hence, the parallels, such as the transition from the impetus theory to Newtonian mechanics, though fascinating, are limited. Another problem that Carey points out is that, although there are similarities, the theoretical structures used in science are, to some degree, distinct from cognitive structures.

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<sup>21</sup> In order to distinguish this "knowledge constructivism" from Piagetian-based constructivism we might term the latter "structuralist constructivism."

Hence, any comparison is naturally strained.

### Theorization and Theoretical Complexity

Flavell (1963) described several shortcomings in the theorization of Piaget:

There is a great deal of vagueness, imprecision, instability of concept definition, and other obstacles to communication in Piaget's theoretical writings . . . .

There is also the related tendency to leave large gaps between theory and empirics, almost to distantiate one from the other . . . . [and a] persistent disinclination to cast his theory in such a form as to make it an instrument of deduction, of hypothesis-generation . . . .

. . . Piaget's bent toward theoretical overelaboration, [is] often bordering on the pretentious. . . .

. . . Piaget sometimes becomes unduly fascinated with theory-construction as an intellectual exercise, as a challenge to his ability to synthesize and analyze, to ferret out hidden logical connections between this theoretic element and that . . .

. . . Also, in reading through one of Piaget's extended chains of theoretical discourse, one often has the uneasy feeling that there is something awry in the logic . . . .

. . . It is as if Piaget were conducting his scientific affairs--doing experiments, interpreting their results, constructing theories, and so on--according to an implicit system of rules rather different from that by which most of his readers play. (pp. 427-429)

Flavell (1963) also faulted Piaget for his tendency to overinterpret; "to state that

such-and-such a datum supports such-and-such theoretical assertion without indicating how it does, or even could, support it;" and "to force unwillingly data into preset theoretical molds" (p. 433).

Vygotsky (1986) criticized Piaget for naively trying to avoid taking a philosophical/theoretical position before presenting his findings, for trying to let the facts speak for themselves, and for presenting his theory as if it followed his findings, when, in reality "the very choice of experiments is determined by hypothesis"; "facts are always examined in the light of some theory and therefore cannot be disentangled from philosophy" (p. 15).

Crain (1988), Hughes (1986), and D. G. Phillips (1995) stated quite frankly that Piaget is difficult to understand. Phillips described how Piaget's ideas are scattered throughout more than sixty books and hundreds of articles some of which have still not been translated from French into English. The focus of much of Phillips' career has been the organization of Piaget's work into a comprehensible, organized program which can be used by teachers for enhancing the development of children's logical thinking. The fruit of this thirty-year endeavor was the publication of *Structures of Thinking: Concrete Operations*.

### Four-Stage Theory of Cognitive Development

Gardner (1983) stated, "While the broad outlines of development as sketched by Piaget remain of interest, many of the specific details are simply not correct. Individual stages are achieved in a far more continuous and gradual fashion than Piaget indicated. . . (p. 20)."

Gelman and Baillargeon (1983) argued that Piaget's work presents an overly strong commitment to the view that all cognition develops through four successive stages each of which is characterized by the emergence of qualitatively distinct

structures. This has led Piaget to ignore or even "summarily dismiss" alternative accounts. Evidence shows that the preoperational child has more competence than anticipated and that the concrete-operational child utilizes concepts in individual domains without using the kind of integrative structures that would be required by a general stage theory. That is, the child can apply a mental structure such as conservation in one domain, such as mass, a year or two before another, such as weight (Flavell, 1963, p. 22). This type of horizontal *decalage*, according to Gelman and Baillargeon, does not indicate that there is no stage-like advance within a domain, but that Piaget's four-stage scheme cannot be applied across all domains for a given individual. What is not denied is that young children's thinking is qualitatively different from that of older children and even amongst children of the same age level; a fact with which the fields of curriculum and instruction have not yet successfully dealt; that is, how to individualize an educational system.

Cohen (1983) questioned the Piagetian view of "a series of distinct stages of development, each marked by the ability to perform certain logical operations and the inability to perform others" (1983, p. 122). According to Cohen the research indicates that there is overlapping between stages, that children and adults can drift in and out of using logical thought, that there is a somewhat looser and less precise organization of operations than that postulated by Piaget's grouping theory, and that adult thinking is much more complex and extensive than portrayed in the descriptions of the formal operations stage.

The finding that the modification of task variables can easily modify a subject's response led Brainerd (1977) to argue that it is "difficult to reconcile the stage viewpoint with large behavioral changes produced by . . . small stimulus manipulations" (429).

Flavell (1963, p. 441) faulted Piaget's four-stage theory for not covering the whole life cycle, for not going beyond intelligence as described in formal operational thought into the study of adult "wisdom" and for not covering such phenomena as attitudes,

beliefs, and judgements (supposedly other than moral judgement which Piaget did investigate).

Cohen (1983) expressed his belief that Piaget overemphasized formal operational thought as the apex of all thinking. He cited studies regarding the way scientists think which indicate that they use patterns of formal logical thought far less than philosophers of science suppose. In his opinion, Piaget left far too little scope for unconscious, imaginative, and creative thought in children and scientists.

Brodzinsky, Sigel, and Golinkoff (1981) supported the position of Neimark who pointed out that if cognitive stages were universal, as Piaget claimed, then all adults would attain the formal operations stage. Many adults never achieve the ability to use formal operational thought; a finding which led Piaget to rethink his position regarding this fourth stage.

Modgil and Modgil's (1982) summary of the critiques of Piaget's stage theory indicated a trend away from the conception of development as a linear progression to one that is more interconnected, branching, and intertwining. In tune with this observation was Novak's (1982) finding that any one subject may exhibit as many as three levels of cognitive operations sometimes within a single interview session.

Tomlinson-Keasy (1982) posited that as more information is acquired, the stage theory issue will diminish in importance and researchers will devote their efforts to charting structural changes in logical reasoning without trying to defend a rigid, discontinuous, stage view of development.

### Notion of Structures

According to Gelman and Baillargeon (1983) the concepts of "structures of knowledge" and "cognitive processes" are now widely accepted. However, not everyone accepts Piaget's views. For example, Kendler's learning-theory view and the

information-processing tradition focus more on the improvement of stimulus encoding, short-term memory, long-term memory, and retrieval. The authors agreed with Flavell that the notion of structures is here to stay, but that Piaget's description of them and theorizing about them needs to be improved.

Brown and Desforges (as summarized in Modgil and Modgil, 1982) stated that research does not support the notion of structures existing independent of content because their generalization is not always possible when the content changes. They expressed a preference for identifying cognitive structures within specific domains which may eventually produce enough information to identify cross-domain generalities.

In response to the above approach, Carey (1987) warned against abandoning the search for domain-general characterizations of cognitive development such as the shift to non-egocentricity, the groupings of concrete operations, distinguishing appearance from reality, and developing cross-domain causal schemata, which capture so well the differences in thinking, for example, between a 4-year-old and a 10-year-old. She cringed at the thought of the field being reduced to "describing thousands of piecemeal developmental advances" (p. 160) with no unifying description. As an alternative to the theories of cross-domain structures and stages, she posited the idea of viewing cognitive development as theory change, the emergence of new theories from previously held ones. Unfortunately, it is beyond the scope of this summary to enter into the details of this fascinating proposal.

Tamburrini (1982) linked the problem of horizontal *decalage* across domains as evidence of the lack of generalizability of certain structures. However, she stated that research supports the notion that, for concrete operational structures, subjects who score at a high level of attainment on Piagetian tasks generalize more than subjects at low levels of performance. For formal operational thought, however, performance is more contextually bound, that is, generalization from a problem in one content area to



another is highly dependent on whether the material is verbal/ symbolic or concrete, on the complexity of the relationships in the problem, and on the subject's background experience.

Gardner (1983) referred to this same problem of *decalage* across domains when Piaget's theory called for the application of operations to any manner of content. "Theoretically related abilities [such as conservation]" Gardner stated, "turn out to emerge at disparate points in time, (p.21)"

### Concept of Equilibration

The equilibration process is "the process of bringing assimilation and accommodation into balanced coordination" (Flavell, 1963). Piaget viewed equilibration as a gradual, continuous, incremental process. He stated, "In brief, no structure is ever radically new, but each one is limited to generalizing this or that form of action abstracted from the preceding one" (as cited in Flavell, 1963, p.240.) Flavell described "a sequence of equilibration state 'moments' within an ongoing, continuous process of equilibration" (p. 263). Different kinds of equilibration states are attained as structures develop and become integrated. The differences are ordered. Each successive state is "better equilibrated"; that is, it attains "a higher degree of equilibrium." Higher states incorporate and integrate "into a broader and more complex totality the elements (cognitive actions) of the lower states without annulling or contradicting them" (pp. 239-240).

As far as this description goes, I agree with it. However, in my opinion, it fails to differentiate between minor and major accommodations. When I observe the "aha!" moments of children when they suddenly become aware of and understand how something works--moments when former conceptualizations are suddenly discarded for the new insight gained--I get the feeling that development is not always a gradual,

incremental process, that there are occasional leaps, sudden paradigm shifts, flashes of structural consolidation that may have been building up slowly over a period of time. This is like the potential chicken which passes from an equilibrated state inside the egg and then, after fertilization and gestation, suddenly breaks into another, higher-level state of equilibration as a new-born chick. Another example is a sand pile to which sand is added grain by grain. The system is stable until suddenly one grain (the proverbial straw that broke the camel's back) causes an avalanche resulting in another altered but stable pile. The procedure of adding one grain at a time to the same pile can be continued indefinitely, but the pattern is the same: incremental growth with periodic major shifts. The process contains stages of near dormancy, phases of slow change, and moments of sudden advance. The idea of such shifts is expressed in the theory of evolution of general systems of Ervin Laszlo (1989) who describes the general direction of historical and natural evolution as the attainment of higher-level systems through "sudden bursts of creativity that come in the wake of critical instabilities in the lower-level systems" (p. 31). Such a view runs parallel to Kuhn's (1962) theory of the history of science in which the gradual buildup of anomalies leads to a sudden reconceptualization which is able to account for the anomalies in a simpler and more elegant manner. In the minds of children, low level structures of magic and animism are used to account for causation. Anomalies may at first be ignored, but, with experience and maturation, they gradually enter awareness and place increasing pressure upon the inadequate structure until there is a sudden shift to a qualitatively, if not "radically" new, higher-level, equilibrated structure which is now capable of assimilating a wider range of data with greater logical consistency and coherence. Because Kuhn and Laszlo were contemporaries of Piaget, I am surprised that more attempts to coordinate their theories was not carried out--especially since Laszlo's work has its roots in physical and biological evolution and Kuhn's work lends support to Piaget's idea that there is a parallel relationship between the ontogenesis of scientific knowledge in

childhood and the history of science. Indeed, as was discussed previously, Carey (1987) proposed that cognitive development be viewed as the emergence of a higher-level theory from a lower-level theory. Such an approach could accommodate the various types of phases: equilibration; slow buildup of new, contradictory information; a sudden, creative, integrative shift; and then a higher level or "moment" of equilibration.

Another aspect of equilibration is its developmental force. Flavell (1963) considered the equilibration process to be the "the propellant for change and transition" which moves "the child through the ontogenetic sequence" (p. 238). He stated that the equilibration model "imputes a certain directionality, even a certain teleology, to ontogenetic development" (p. 239). Consider, for example, the statement of Piaget and Inhelder (1969): "In an act of intelligence . . . the end is established from the outset and pursued after a search for the appropriate means" (p. 9). I find equilibrium to be a very weak "propellant" for development. It is too close to the notion of homeostasis which, in my opinion, gives a poor account of the evolutionary advance into novelty (Whitehead, 1978). Equilibration may explain the mechanism for how the mind responds to and processes experience, but it tells a meager story of how most children are motivated to penetrate the unknown aspects of their environment. I find that the concepts of "competence motivation" (White, 1959), "final cause," and "subjective aim" (Whitehead, 1978) to make much more sense. When I observe children, I sense in their intentions a natural and logical striving to understand and to gain control over their world—the physical, social, psychological, and spiritual environments with which they interact and their own self. On the other hand, I find the idea of seeking to achieve equilibration through accommodation to be adynamic. I can understand why, if a child was in a state of disequilibrium, he/ she would want to regain equilibrium. But, if a child's mental structure is in a state of equilibrium, from whence comes the motivation to interact with the environment—especially if he/ she has already attained the ultimate

aim of development--a state of adaptation, as temporary as it may be?

In *The Psychology of the Child*, Piaget and Inhelder (1969) came close to admitting the inadequacy of equilibration to explain behavior by momentarily considering affective forces as possible developmental propellants:

It may even seem that affective, dynamic factors provide the key to all mental development and that in the last analysis it is the need to grow, to assert oneself, to love, and to be admired that constitutes the motive force of intelligence, as well as of behavior in its totality and in its increasing complexity. (p. 158)

But then they return to their discussion of their preferred mechanism -- equilibration.

### Maturation Factor

Other than indicating that maturation is one of the main factors which influence development—along with physical experience, logico-mathematical experience, social transmission, and equilibration--Piaget has nothing much to say about the specific role of physical maturation (Ginsburg, 1988, p. 251). More recent investigations of brain development are shedding more light on this aspect of human development.

### Nativism versus Constructivism

A drawback of Piaget's theory discussed by Gelman and Baillargeon (1983) is his insistence that, because structures are actively constructed by the child, they cannot be considered as innate. However, there is much evidence that at least some structures which underlie knowledge systems are innate, for example, counting and

language. Evidence indicates that children are able to draw upon a "wired-in," spontaneous tendency to count and that they have specific neural machinery for computing phonetic representations of the spoken language that they hear.

Beilin posited that research supports neither Chomsky nor Piaget's position. He stated that, "the theoretical relation between cognition and language is one of partial autonomy for each system [Beilin, Chapter 8, p. 120]" (cited in Brodzinsky, Sigel, & Golinkoff, 1981). That is, cognition and language have certain qualities that are unique, have other characteristics in common, and influence one another's development. Beilin argues that in spite of the more recent emphasis given by the Genevan School to language as a source of knowledge, they have not yet been able to account for the research indicating that linguistic activities can induce operational structures; that language is not just a vehicle for the expression of thought, but that it is also "a dynamic force that promotes further development in intellectual and social domains" (p. 11).

Gelman and Baillargeon (1983) reviewed Piaget's 1975 debate with Chomsky and Fodor which began with a discussion of language acquisition and ended with a broader argument about the constructivism of Piaget versus the innatism of Chomsky. The biologists who were present supported the idea of innatism, but, as the authors point out, one position does not necessarily contradict the other.

The authors presented an example of innatism from ethology in process terms similar to those of Whitehead (1978), Jordan (1981b), and Streets (Jordan & Streets, 1972). The biologists explain that it has been found that the young white-crowned sparrow must be exposed to the song of adult white-crowned sparrows during its first 10 to 50 days of life. Otherwise, the song is basically the same but noticeably distorted. The ethologists argue that there is a *critical period* for hearing the song; that the white-crowned sparrow is born with a *template* for the basic song; and that experience serves to fine tune the template:

The idea is *not* that development involves a bit of innate structure and a bit of learning but *that development is a function of the organism's interaction with its environment*. The potential for structural change is not reached unless there is development, that is, an interaction between structure and environment. Nevertheless, the potential is innately given. (p. 218)

From a "process-as-reality" point of view cognitive structures could be considered to have two forms--non-actual and actual. The non-actual form is "mental structure as potentiality" which has a predetermined, yet open-ended form which is more advanced than, for example, that of a white-crowned sparrow or that of a chimpanzee, but less advanced than, for example, the omniscience of God and His Prophets. It is human in its nature, form, possibilities, and limitations.

Piaget does not appear to acknowledge the concept of potentiality as a non-actual form of reality in relation to cognitive capacities. Consider these statements:

An internal mechanism (though it cannot be reduced to heredity alone and has no preestablished plan, since there is in fact construction) is observable at the time of each partial construction and each transition from one stage to the next. (Piaget & Inhelder, 1969, p. 157)

In general, if we are to account for the biological roots of these structures and the fact that they become necessary, we must think in terms neither of the exclusive action of the environment nor of an innate preformation, but of self-regulations functioning in circuits and having an intrinsic tendency towards equilibrium. (1972, p. 60)

Jordan (1972), on the other hand, avoids the pitfall of the antiquated notion of

innate preformation. He recognizes potentiality as a non-actual form of reality and identifies the actualization of potentiality as a "first principle" underlying the nature of reality, of development, and of education.

The "actual" form of a mental structure is the manifest form which is brought about by the child's interaction with the environment. It is the form which is constructed by the child through learning, either unconsciously or consciously. Without interaction the mental structure is not actualized, that is, it does not develop. It remains dormant in the non-actual realm of potentiality. Some potentialities can be actualized at any time during the life cycle. For others there are critical or sensitive periods. For example, the sensitive period for learning a foreign language without an accent is from 2 to 10 years of age. Afterwards, it is extremely difficult to acquire a foreign language and nearly impossible to do so without an accent (Jordan, 1981b). In summary, the process view accommodates both the nativist and the constructivist positions.

### Narrow View of Cognition and Logic

Cohen (1983) stated that Piaget tried to "convince psychologists that he had a general theory of child development" (p. 83) when, in reality, he only had an overly narrow and one-sided theory of the development of thinking which inappropriately elevated cognition as the driving force behind emotional, social, moral and language development. He cited as evidence Piaget's lop-sided clinical observations of his own children which are too highly intellectual and devoid of any human interaction and humor.

Cohen also found Piaget's theory of formal operations to be "too logical." After citing one study showing that only a small percentage of British undergraduate students could apply formal operational thinking to a task and another which found that a group of brain-lesioned patients performed better on a logical task than did

undergraduates, Cohen expressed his view that Piaget's expectation of developing formal operational thinkers is unrealistic and that abstract tasks are difficult for those who do not have a logical, philosophical bent.

Glick pointed out that there is not just one form of rationality but, rather, multiple forms which are elicited and maintained by a person's particular sociocultural environment. One form is not necessarily more primitive or advanced than another as long as it leads to effective adaptation within the cultural system (Brodzinsky, Sigel, & Golinkoff, 1981).

One type of cognition which is underestimated by Piaget is intuition. He speaks enthusiastically of "the victory of operation [logic] over intuition" (1965, p. 149) implying that intuition is an inferior and fallible means of knowing. However, there are instances in many people's lives (including scientists) when decisions based on intuition have been found to be more viable than if they had followed a course of action based solely on logic. Intuition needs to be acknowledged as a valid means for acquiring knowledge and people need to be trained to become consciously aware of their intuitions and to learn to differentiate inspired intuition from vain imaginations.

Reid faulted Piagetian theory for neglecting the study of the vital relationship between affect and cognition and discusses the role of emotional distress in disturbing proper intellectual functioning (Brodzinsky, Sigel, & Golinkoff, 1981).

Brain-based research supports this concern of Reid by confirming that incoming stimuli and information is first processed by the emotional centers of the brain before being processed by the cognitive, logical centers (Caine & Caine, 1991; Goleman, 1995; Sylwester, 1997).

### Notion of Intelligence

Besides being criticized for having a narrow view of cognition, Piaget has also



been faulted for maintaining a too limited notion of intelligence. Cohen (1983) pointed out that in realms such as art, music, and literature there are types of thinking that are utilized which go far beyond the narrow, formal logic emphasized by Piaget. He proposed that a theory be developed which places logical thinking in its proper place, along with other styles of thinking, in the context of a child's life as a whole.

Gardner (1983) appears to have been thinking along the same lines as Cohen as evidenced in his *Frames of Mind: The Theory of Multiple Intelligences* in which he goes beyond Piaget's focus on logico-mathematical cognition by positing the existence of seven types of intelligence: linguistic intelligence, musical intelligence, logical-mathematical intelligence, spatial intelligence, bodily-kinesthetic intelligence, interpersonal intelligence, and intrapersonal intelligence. More recently (Checkley, 1997) Gardner added natural intelligence which he defined as the human ability to discriminate among living things (plants, animals) as well as sensitivity to other features of the natural world (clouds, rock configurations). He is also contemplating the inclusion of a ninth intelligence which he refers to as existential intelligence that "refers to the human inclination to ask very basic questions about existence. Who are we? Where do we come from? What's it all about? Why do we die?" (as cited in Checkley, 1997, p. 9)<sup>22</sup>.

Regarding Piaget, Gardner (1983) both found fault and gave praise. His acknowledgements of Piaget's contributions to three areas of psychology are presented in the last part of this essay. On the negative side Gardner (1983) stated, "In

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<sup>22</sup> It is of interest to note that Goleman (1995), although he made no critique of Piaget's limited notion of intelligence, expanded upon Gardner's "intrapersonal intelligence" by drawing together the relevant research from the biological, sociological, and psychological sciences and identifying a possible tenth area of the human intellect which he termed "emotional intelligence." Goleman described emotional intelligence as having five dimensions: self-awareness, handling emotions generally, motivation, empathy, and social skills.

my view, Piaget has painted a brilliant portrait of development in one domain--that of logical-mathematical thought--but has erroneously assumed that it pertains to other areas, ranging from musical intelligence to the interpersonal domain" (p. 133). While acknowledging Piaget for having illuminated "that form of human intellectual growth which is valued most highly by the Western scientific and philosophical traditions (p. 20)" Gardner further faulted him for having "avoided forms of knowledge that are simply memorized (like word definitions) or restricted to certain cultural groups (such as those that favor 'high' art)" and for having ignored "the steps entailed in achieving other forms of competence--those of an artist, a lawyer, an athlete, or a political leader" (p. 20). He further stated that "Piaget tells us little about creativity at the forefront of the sciences . . . [discovery of new phenomena or the positing of new problems] or other realms of human creativity " and that his tasks are "fairly remote from the kind of thinking in which most individuals engage during their normal daily lives" (pp. 20-21).

#### Lack of Attention to Individual Differences

Cohen (1983) faulted Piaget and Piagetians for not identifying the individual, social, and economic differences which impede or enhance logical thought. He found that there is a dearth of papers which show exactly how Piaget's theory accounts for the clearly established, individual and cultural differences in cognitive development. According to Cohen, because Piaget was looking for universal norms rather than examining individual differences, his description of precisely how development takes place is far too vague; that the concept of "experience" is far too unspecific; and that much more investigation and explanation needs to be done to connect the research on personality and styles of thinking to Piaget's general theory of intellectual development. (See also Brodzinsky, Sigel, & Golinkoff, 1981, pp. 13-14, and Ginsburg, 1988, pp. 255-256 for similar arguments.)

### Vagueness of "Interaction" Concept

For Cohen (1983), Piaget's concept of "interaction" is too limited and sketchy. Piaget emphasizes the interaction between the genetic base and the environment, but his concept of environment is far too limited to physical "things" to the exclusion of the social environment. Also, Cohen faulted Piaget for considering too frivolous the possibilities of investigating how the manipulation of different environments could lead to different rates of cognitive growth. Cohen summarized Piaget's vague account of "interaction" as occurring "as the child bumped and banged about the world, received feedback, and acted on its mistakes . . . and these act as a spur to remodeling its views of what is real" (1983, p. 95).

Feuerstein (1980) stated, "Although Piaget conceives of cognitive development as a function of an interaction between the organism and the environment, the environment is conceptualized as a domain of objects" (p. 15). Feuerstein, building on the work of Vygotsky gives more importance to the role of "social interactions": "Whereas Piaget's approach is conceptualized in his stimulus-organism-response (S-O-R) formula, the theory of MLE [mediated learning experience] may be expressed by the formula S-H-O-R, in which a human mediator (H) is interposed between the stimulus and the organism" (p. 16).<sup>23</sup>

Vygotsky (1986), as will be discussed further in the section below on "egocentric speech," gave more importance to the social environment; the enhancement of communicative, egocentric, and inner speech via social interactions; and the

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<sup>23</sup> What needs to be clarified in Feuerstein's formulation is that the human (H) can be both the stimulus and the mediator (H-H-O-R). That is, the environment is composed not just of objects but also people. An example in education would be the situation in which a teacher mediates (or guides) the interaction between the class and a guest speaker.

development of higher levels of thought via inner speech. Kohlberg (1987) stated, "Vygotsky recognized crucial aspects of linguistically mediated social (i.e., interpsychological) and individual (i.e., intrapsychological) functioning that Piaget failed to recognize" (p. 219).

From a holistic point of view, I would extend Cohen's criticism of Piaget's concept of environment. In addition to ignoring social experience, Piaget's psycho-mental world is limited to logico-mathematical experience--reflecting on one's actions on objects. This world of reflection and thought is inhabited by far more entities with which to interact: memories, dreams, daydreams, fancies, imaginations, forms, ideas, ideals, goals, possibilities, hopes, fears, imaginary people and places, and many others.

Also, Piaget's concept of environment excludes spiritual entities: God, the Holy Spirit, Prophets, and human souls in this world and beyond. A person's interaction with these entities constitutes spiritual experience. Crain (1985) stated, "At 15, Piaget experienced an intellectual crisis when he realized that his religious and philosophical convictions lacked a scientific foundation" (p. 88). His later studies included religion along with philosophy and logic (Flavell, 1963; Ginsburg, 1988). However, his work does not attempt to integrate religious concepts with intellectual theorizing and experimentation. From a holistic point of view, such a fractured and limited view of human nature and development is inadmissible because it leads to the creation of fragmented and limited programs in fields such as socioeconomic development, health care, and education.

Another environment which is barely implicit in Piaget's concept is that of the self--a person's interaction with and dialogue with the self. Jordan and Streets (1973a) identify the self as the most pervasive environment in the life of a human being with inescapable interactions occurring twenty-four hours a day.

## Methodology

Cohen (1983) stated that in order to understand the minds of children they need to be studied in their natural habitats. He faulted Piaget's clinical method for removing the child from everyday situations. He observed that even in the few cases when Piaget did observe children in a home setting, the emergent picture is too devoid of other people; that he overemphasized children's relationship to things; and that he needed to devote more effort to "the child's conception of people" in order to produce a more well-rounded theory of intellectual development. Piaget viewed social life as a product of reasoning when much is to be said for the view that reasoning may develop out of social life.

Another critical issue pointed out by Cohen (1983) is Piaget's failure to seek alternative explanations for his findings. He found it unacceptable that Piaget should emphasize the importance of logic and the hypothetico-deductive method while failing to eliminate false hypotheses. Cohen criticized Piaget for repeating experiments such as the conservation of liquid amount without changing the fundamental conditions very much. He would have liked Piaget to have manipulated variables such as the vocabulary used; the degree of familiarity of the testing situation; the emotional state of the child; and the use of demonstrations, explanations, and training--all of which have been shown to have an impact on task assessment.

Crain (1985) pointed out that Piaget often "violates the canon of standardized interviewing" (p. 89) by occasionally changing his interview questions during an interview in order to pursue a deeper understanding of the child's thinking. Other researchers have found this to be a strength of his clinical method (Fincham, 1982; Ginsburg, 1988, p. 246). D. G. Phillips (1996) utilized a moderate approach. He formulated uniform interview protocols to provide consistency amongst researchers,

yet he encourages the use of flexible pursuit questions to verify a child's reasoning.

In the present study, the only drawback I sensed with "pursuit questions" such as: "It sure looks like there are more in this row. Are you sure that they are the same?" was that young children who have a near reverence for authority figures may give in and agree with the adult even though their intuition or knowledge may be correct. It takes a great deal of self-confidence and bravery to stand up to the counter arguments of an authority figure. On the positive side, however, teachers who incorporate the clinical method into their ongoing, formal, and informal assessment program have the special advantage of building rapport and a spirit of inquiry over a period of time; an atmosphere in which it is acceptable to counter the statements of adults. The occasional investigator is more likely to be perceived by young children as an imposing, unknown, authority figure that should be treated with caution.

Hughes (1986) stated that "it is felt that Piaget's tasks frequently do not make sufficiently clear what the children are supposed to do" (p. 19). This is a legitimate complaint which, in my opinion, has been largely corrected by D. G. Phillips (1996) and his collaborators who have refined and clarified the tasks, the procedures, and the instructions.

Flavell (1963, p. 430-431) lamented the fact that Piaget did not report his results in a scientifically acceptable way; that he failed to explain exactly what he did in the experiments: materials, protocol, testing conditions, sample size, sampling technique, ages and background of subjects, etc.; and that he did not report any quantitative data or carry out any statistical analyses. D. G. Phillips (1996), his co-researchers, and other investigators who have carried out follow-up studies (Flavell, 1963, p. 431) have corrected in their work this weakness.

## Development and Role of Language

Flavell (1963) explained that there always has been and always will be a problem of language ability in developmental studies of the Piagetian type. The meaning that a child gives to the words in a question and the correspondence between the child's verbal reasoning and his/ her cognitive structures can never be known for certain. The extent of a child's vocabulary can lead one to over- or underestimate the level of cognitive functioning.

According to Berko and Brown, "Piaget is inclined to see through words as though they were not there and to imagine that he directly studies the child's mind" (as cited in Flavell, 1963, p. 437).

Another aspect of language concerns its development and its role in cognitive development. According to Cohen (1983) Piaget emphasized the infant's motor coordination as the basis of language development, thereby relegating the role of adults to a secondary status. He criticized Piaget's view that young children's prelinguistic communication is solitary and that there is no true communication with others. He cited studies which show the existence of rich interactions between infants and their mothers based on voice, words, tone, and facial expressions. According to Cohen, Piaget saw children as "maturing" into language and he contrasted this with Chomsky and Fodor who consider language to have an "innate fixed nucleus" (1983, p. 141) in the brain and language development to depend on the child's having social interaction of at least a minimal quality. Cohen, however, agreed with the more interactionist view of Piaget but he placed more emphasis on the quality of the social interaction with adults than on motoric interaction with the physical environment.

## Concept of Egocentrism

In my understanding, Piaget's concept of egocentrism has two aspects--social and physical/ perceptual--both of which have been criticized and confused. Flavell (1963) defined egocentricity as "a cognitive state in which the cognizer sees the world from a single point of view only--his own--but without knowledge of the existence of [other] viewpoints or perspectives and, a *fortiori*, without awareness that he is the prisoner of his own" (p. 60). This definition does not clearly differentiate social and physical/ perceptual viewpoints. Piaget gave a clearer distinction via two separate definitions:

[It is the] undifferentiation between the other and the self.

It consists only in taking as sole reality the one which appears to perception (as cited by Butterworth, 1987).

Regarding the social aspect, Cohen (1983) referred to Piaget's findings that children seldom play together before the age of four and a half and that they do not have two-way conversations before five and a half. He cited studies in which two-week-old infants respond to the facial expressions of others. Other studies found early responses in infants to the voice and facial expressions of mothers. He even examined various interpretations of the peek-a-boo game which show subtle social skills. Unfortunately, Cohen implied that these findings also negate physical/ perceptual egocentrism. The cited studies do not indicate that the infant was "seeing" things from the point of view of the adult. The point that Cohen seemed to be trying to make is that even infants are very social beings who learn a great deal from social interactions, and that Piaget seemed to have neglected this aspect of human nature.



In relation to physical/ perceptual egocentrism Cohen (1983) cited studies which found that under very special experimental conditions even very young children of 2 to 5 years showed understanding of perspectives other than their own present point of view. Butterworth (1987) cited similar studies. As will be mentioned in my comments below on object permanence, this only suggests that the onset of the child's escape from physical/ perceptual egocentrism perhaps has earlier beginnings than those found by Piaget. Nevertheless, the studies do not negate Piaget's findings about children's ability to visualize objects as seen from a different physical/ perceptual perspective. In my own elementary school science laboratory I have administered the Piagetian tasks which relate to this type of egocentrism to students in grades 1-5 and the results are in accord with those of Piaget.

### Egocentric Speech

Vygotsky (1986) criticized Piaget's view that egocentric speech of children, i.e., self-talk which is directed to no one, which expects no response, and which is incomprehensible to others, "appears as a product of the child's activity, as a stigma of the child's cognitive egocentrism ..." and is "therefore, useless ..." playing "no essential role in child behavior. . ."; "a symptom of weakness and immaturity in the child's thinking, a symptom that must disappear in the course of child development" (pp. 28-29). Vygotsky set up experimental tasks for children in which were added "a series of frustrations and difficulties" (p. 29). For example, a child might be required to draw but find that there was no paper or pencil. He found that in this type of situation egocentric speech increased and played a useful role in the child's attempt "to grasp and to remedy the situation" (p. 30). Other experiments showed that egocentric speech had a directing and planning function in children's problem-solving activities.

Contrary to Piaget's developmental sequence of autistic nonverbal thought

moving through egocentric thought and speech and then on into socialized speech, Vygotsky (1986) posited a sequence of development which views speech as social in nature from the very beginning, then moving into egocentric speech which guides thought processes and finally into inner speech which also enhances thought. Hence, where Piaget saw egocentric speech disappearing and giving way to socialized speech, Vygotsky saw egocentric speech splintering off from socialized speech and going "underground" into inner speech. Having no direct empirical evidence of this Vygotsky relied on indirect, verbal, "thinking aloud" reports of inner speech which he found to have the same characteristics as egocentric speech.<sup>24</sup>

### Late versus Early Developing Structures

Piaget often stated the approximate ages or stages at which mental structures were developed based on the results of the tasks which he and his coworkers administered to thousands of children over the course of many years. It has been found that the variation of several factors influences the manifestation of a given structure which usually ranges from a primitive, prescribed domain of application to a more sophisticated and generalized utilization of the structure: using small quantities in number tasks; using familiar objects and situations in the protocols; increasing the

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<sup>24</sup> Although oversimplified, I found the following line of reasoning in Vygotsky's work as a consequence of this finding regarding the importance of inner speech. Inner speech enhances thinking and problem-solving. Speech is social in nature from its very onset. Therefore, enriched social communication can enhance inner speech which will, in turn, facilitate the development of thought. Hence, "thinking out loud" with a child can help the child to reason. Vygotsky's subsequent finding that some children could perform better with social-verbal assistance than they did alone and that some children benefitted more than others by this type of assistance, led to his creation of the theory called the "zone of proximal development": "the discrepency between a child's actual mental age and the level he reaches in solving problems with assistance" (Vygotsky, 1986, p. 187).

child's knowledge of the world; improving the child's information-processing capacities; developing perceptual and cognitive strategies (Gelman and Baillargeon, 1983); training through presentation of rules, feedback, modeling and discrimination learning sets; social interaction with a competent peer; using facilitative wording of questions; varying the social condition of the assessment situation (Siegler, 1995 & Cohen, 1983); studying children in contexts that make sense to them, such as when playing simple games (Hughes, 1986); the child's awareness of the nature of the task and the meaning of the vocabulary (Cohen, 1983); the number of objects involved in the task; the number of differences between objects; the total information load; the number of dimensions to be coordinated; and variations of properties other than the ones on which the subject needs to focus (Kingma, 1984). For example, Goswami and Brown (1990) found evidence of analogical reasoning, which Piaget places at the formal operations stage which begins around 12 years, in children of 3-4 years of age. The factor which they varied was the familiarity with the domain. The Piagetian analogies, they explained, were too difficult and unfamiliar. Their results, nevertheless, showed great differences amongst 3- to 6-year-olds with a pronounced developmental spurt occurring during the three to four year period.

Siegel and Hodkin (1982a) also faulted the Piagetian system for ignoring the multidimensional nature of tasks and failing to control for factors such as language, perception, attention, social relationships, and memory.

Wadsworth (1984) and Cohen (1983) cited studies, in which, contrary to Piaget's finding that children demonstrate object permanence between the ages of 8 and 10 months, infants of only 3 months seemed to be convinced that objects which have been hidden continue to exist.

In relation to the abilities of infants and the object permanence concept, a far more profound criticism referred to by Cohen is the child's concept of the mother and father. Can they be considered objects? For the infant, does the mother really cease to

exist when she is out of sight? Can there be an unconscious sense of "mother permanence"? In the case of unwanted pregnancies, can the infant feel that the mother is "psychologically absent"? I find these to be thought-provoking questions worthy of investigation.

Cohen stated that Piaget stressed "the inability of children to form genuine concepts" and that they "could not move from the particular to the general" (p. 111). He then cited studies which show that in special testing situations children as young as two to four years showed evidence of class inclusion when familiar concepts are used in the tasks.

Again, what an educator looks for is not just isolated signs of a budding structure, but the ability to apply the structure to varied and new content required in the curriculum. This ability appears later than the ages of the subjects referred to by Cohen.

Cohen (1983) also cited studies which indicate that by changing certain experimental conditions children demonstrate transitivity and iterated measurement sooner than indicated by Piaget. This is a legitimate criticism of Piaget; that he needed to test students under a wider variety of conditions. This fault, however, in my opinion, has been corrected by the Developmental Activities Program of D. G. Phillips and D. R. Phillips (D. G. Phillips, D. R. Phillips, Moore, & Melton, 1994) in which children are observed under natural conditions using a wide variety of familiar objects chosen by the children with interview questions being phrased in various ways. This is combined with formal, Piagetian-based tasks (D. G. Phillips, 1996) which serve as milestones of development and can be used to track a child through a cognitive curriculum. Having data collected on various populations using the same tasks provides a firm, scientific foundation for comparative analysis for both individuals and groups and for the programs which are designed to enhance the development of logical thought.

As with so many critiques which find an earlier demonstration of a mental

structure, as an educator, I find the results to be of little help for practice. I have looked at the findings of hundreds of studies regarding concrete operational cognition. Only in a very, very few did I find that 100% of the subjects passed any particular task, and these were usually older adolescents performing tasks for structures which are normally developed much earlier. Also, with the exception of James Fodor (1975) who has claimed that all of the concepts that we could ever have are innately given, and only triggered or accessed by certain learning experiences (the misguided educational implication of which is that you can teach any concept at any time) I have found no investigator who has claimed that all children have any or all structures formed from birth, much less from conception. Piaget never claimed that structures were highly age-related. Some adults never form some of the higher-level structures. Hence, as an educational practitioner, one is always faced with the necessity of diagnosing the developmental levels on an individual basis and it would be poor practice to make a judgement based on only one task performed with only one set of materials and one set of questions in a formal interview setting. "Diagnosis" means getting to "know" your student and his/ her level of understanding. True understanding of a child's grasp of a concept, such as conservation, should be demonstrated in a variety of settings, both natural and formal, with a variety of materials, and in response to a variety of questions using various vocabulary terms. Based on such a diagnosis the teacher can then prescribe the appropriate educational activities and questions which will neither bore nor overly frustrate the learner, but, rather, will allow him/ her to actualize his/ her potentiality at an optimum rate, that is, not being held back or forcefully accelerated.<sup>25</sup>

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<sup>25</sup> This idea of the proper fit between the learner's present level of development in a particular domain and the learning experience was referred to by Jordan (1981b) as "optimum disparity." Vygotsky's (1986) approach to this idea was more highly refined. He differentiated between the child's ability to grasp new concepts alone and grasping them with assistance. The latter he referred to as the "zone of proximate development": "the distance between the actual developmental level as determined by independent

What Cohen and other investigators seem to be calling into question is not Piaget's finding that children do not think like adults, but rather the genesis of a structure, that a structure has humble beginnings much earlier than what Piaget found under certain experimental conditions. Certainly these early signs of competence show future potential. But the story is not complete by focusing only on precocious performances. This can lead curriculum planners to "push down" curricular concepts. Then, when the students whose structures are not well formed encounter less familiar materials and problems, transfer is more difficult. Research shows that structures continue to grow in strength and generalizability with experience and maturation. It seems that there is never a time when a structure is always perfectly mastered and easily applied to all situations. There are certain circumstances--when content is especially new, complex, or abstract-- that even our best-formed structures are challenged and sometimes strained beyond their capacity (D. G. Phillips, 1995). Hence, the picture of a structure that I am constructing from the research is one in which the mental operations have a much longer history than that painted by Piaget. But the finding that there are wide differences between age levels and amongst individuals and that there are phases of significant advance remains unshaken and needs to be addressed by any educational system that aims at a more personalized approach.

Cohen summed up the critiques of Piaget's findings regarding the age of the appearance of certain structures by stating that Piaget underestimated the abilities

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problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers." (pp. 85-86). Feuerstein took this idea even further by developing the concepts of cognitive modifiability via learning mediated by a trained adult. Feuerstein also developed the Learning Potential Assessment Device which focuses on the process of learning while receiving the assistance of a human mediator with the aim of ascertaining the learner's

potential for modifiability (Feuerstein, 1980; Schneider, 1987).

of young children and that he gave too much emphasis to children's interactions with objects rather than with people. According to Cohen we should say that the application of certain structures by young children is often "difficult" but we should not say "impossible."

### Class Inclusion Task

In *Children and Number: Difficulties in Learning Mathematics* Martin Hughes included a discussion of class inclusion in a chapter titled "Piaget under Attack" which disputed Piaget's notions regarding the development of mathematical logic (which he confuses with children's competence with the mathematical symbol system).

At one point Hughes complained that in Piaget's class inclusion task children "tend to focus on the perceptually salient differences between the sub-groups, while the wording of the problem requires them to focus on the distinction between sub-group and overall group" (p. 21) which requires "disembedded" thinking, i.e., thinking about the language used by the adult independently from the objects embedded in a concrete context before their eyes. Somehow he thought that this is unfair for the child. On the contrary, this is exactly what the task is assessing: Can the child use logical necessity to triumph over misleading, salient, perceptually visible features? If so, he/ she has constructed the class inclusion structure. If there is a problem with the child's level of understanding and familiarity with the language and the terms used, then the results will be affected, but this is a different problem, the solution of which can be found through more careful interviewing of the child.

Hughes (1986) cited an experiment which "involved a teddy bear, a set of flat counters, a teddy-size chair and a teddy-size table."<sup>26</sup> These were laid out so that there

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<sup>26</sup> In relation to his critique of the Piagetian class inclusion and conservation of number tasks, Hughes stated, "According to Donaldson, thinking which is embedded in a context which makes sense comes relatively easily" (p. 21). But the problem is that

were four counters leading from the teddy to the chair and a further two counters from the chair to the table" (p. 19). The interviewer then asked, "Are there more steps to go to the chair or more steps to go to the table?" Two thirds of a group of 3- to 5-year-olds answered correctly. He then stated that this refuted Piaget's claim that children below 7 years typically do not succeed on the class inclusion tasks. Apparently Hughes and the investigator were not familiar with the class inclusion structure which involves the comparison of a superordinate and subordinate classes. Comparing steps with steps does not accomplish this. The number of countable steps between one object and another is not a true class. Classes have extensive qualities which go beyond what is visible, e.g., a particular group of animals and fish could represent a class and a subclass, but not all members could be brought together. The more-than/ less-than relation is non-quantitative. This does not hold true for the number of steps to a chair from a given point versus the number of steps to a table from the same point and along the same path. "Steps" is a class but "the number of steps to the chair" is an extremely limited class the number of members of which could be changed by a simple displacement of the chair. True classes are independent of the spatial-temporal dimension. They are logical in nature. The perceptual cue of distance in Hughes' example is so strong that one could hardly say that logic is being assessed. Comparing distances and end points, rather than classes, could produce the right answer. (Personally, I was more amazed that one third of the subjects answered incorrectly!)

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different contexts have a different sense for different children. The teddy bear task cited here may make sense for British children but I have never seen such objects in the villages of Colombia nor Honduras and I doubt whether anyone has ever seen teddy bears walking to chairs and tables. However, I have seen children in the United States, Honduras, and Colombia who play with objects and who like to arrange them and count them. Hence, the use of counters for the Piagetian task seems to me to be a more universal context which would make sense to most children. Also, for the class inclusion task in this study, for assessing coastal children, I used crabs and fish because these are more commonly seen here than the Iowa farm animals--pigs, horses, and cows--used by Phillips, et. al. in their tasks.



Also, the protocol, as reported by Hughes, failed to ask the child for an explanation of the answer. Hence, many children could have simply used counting instead of some type of logical necessity as required by the Piagetian task.<sup>27</sup>

I cite this type of critique as an example of the superficial critiques made by some authors whose aim apparently is to disprove and then discard Piaget in order to set forth their own views rather than to search for the truth.<sup>28</sup>

### Collections Task

As with class inclusion the criticisms of the pre-classification, collections task revolved around language factors which inhibit or facilitate the child's grouping ability (e.g., Markman, 1978; Markman & Seibert, 1976) and the effects of training (e.g., Raven, 1970).

### Ordering Task

#### Critiques of Piaget's findings regarding ordering abound in the present study's

<sup>27</sup> A more productive line of inquiry would be to compare the success of children on the Piagetian class inclusion task used in this study with a similar task which included no objects. The protocol might go something like this: Are ants animals? Are horses animals? Do you think there are more ants or more horses or the same amount of horses and ants in the world? Why? Are there more ants, more animals, or the same amount of ants and animals in the world? Why?

Assuming that the child has some background knowledge of these animals and with no possibility of using perceptual cues, which would appear first, class inclusion with objects or class inclusion without objects?

<sup>28</sup> This same author went on to cite an experiment in which three-year-olds conserved number when only three objects were used. As was pointed out in the present study, small numbers (1-5) are considered perceptual numbers and do not test the logic underlying the conservation of number structure. To conclude that this one experiment totally undermines Piaget's theory is simply poor scholarship.

review of the literature. Most of the critical results indicated an earlier onset of the ordering structure than what Piaget found (e.g., Clark, 1983; Kingma, 1983b; Koslowski, 1980; Leiser & Gillieron, 1990; Pasnak, 1994).

However, as was pointed out in the review, all of the procedures in these studies varied significantly from those of Piaget (1965), Inhelder and Piaget (1969), and D. G. Phillips (1996). They either trained performance (which has been criticized because competence and understanding are not necessarily attained); included no insertion of rods, or, the insertion of only one rod instead of three; gave the subject a baseline cue; or, did not require the subject to find correspondence between items in two seriated sets.

### Conservation Task

Research in conservation of number and liquid amount were examined by Cohen (1983). He referred to studies in which these occur at earlier ages under certain experimental conditions. One fascinating study found that when children instead of the interviewer poured the liquid high percentages of subjects passed the task.<sup>29</sup>

Murray pointed out that there is so much asynchrony in the development of the various types of conservation (mass, liquid, weight, length, distance, area, and perimeter) and so many nonstructural factors such as task characteristics (type of stimulus material, degree of stimulus abstractness, type of stimulus transformation, etc.) and organismic characteristics (attentional factors, memory capacity, cognitive style, etc.) that the stage-based explanation of the phenomenon loses its explanatory power (Brodzinsky, Sigel, Golinkoff, 1981).

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<sup>29</sup> I tried this recently in my science laboratory with conservation of amount. I had the children transform clay balls into sausages and ropes. I still found many non-conservers amongst first, second, and third graders.

Forman (1993), in his discussion of social constructivism discussed the phenomenon of "social marking" in relation to conservation of liquid amount:

Children do better when asked if the amount of water remains the same after an irrelevant transformation when the task is contextualized as sharing with a friend. Children do less well when given the same task in an objective, nonsocial context. The social symmetry between friends and the social norm that one should endeavor to share equal amounts positions the child cognitively to give serious consideration to the difference between appearances and actuality. (139)

Forman cited the position of Doise and Mugny who argued that these "social markers" are more than motivators for more attentive thinking; that they enter the cognitive process as information tags which facilitate the construction of conservation. Hence, the dynamics of working alone with physical objects is qualitatively different from those which occur during social interaction (139).

Gardner (1983) stated, "There is now evidence that children can conserve number, classify consistently, and abandon egocentrism as early as the age of three--findings in no way predicted (or even allowed) by Piaget's theory" (p. 21). Unfortunately he did not cite any particular studies and apparently never carried out any of his own research with Piagetian tasks. As this author has discussed elsewhere in this dissertation the successful performance of three year olds is highly fragile, confined to very limited conditions, and has not been shown to be generalizable to standard Piagetian tasks.

### Effects of Training

As was discussed above and in the review of the literature, researchers have

found that, through training, precocious performance can be obtained from children of ages much younger than Piaget had found. Explanation, feedback, and various means for altering the task conditions were discussed by Cohen (1983). Yet he himself cited the caveat of Gelman:

There are many occasions when their potential brilliance fails them. Theirs is a competence that is fragile, that can be on again, off again, that is used only in restricted settings, that does not generalize readily.<sup>30</sup> (p. 120)

And it is precisely this ability to generalize a structure which is the hallmark of understanding (Ellsworth and Sindt, 1994). For example, Inhelder, Sinclair and Bovet carried out a training experiment designed to develop class inclusion and conservation skills in children. The trained subjects showed progress during training, retained the progress during a first post-test, but regressed after a second post-test. They write,

It seems that regressions occur when the subject only momentarily establishes certain co-ordinations suggested by a specific situation: his reasoning seems strictly local, cannot be generalized to other situations and is probably not accompanied by the feeling of logical necessity that is another characteristic of a truly operatory construct

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<sup>30</sup> This finding may be related to Vygotsky's (1986) concept of the "zone of proximal development" in which it is acknowledged that a child can perform at a higher level with the assistance of an adult or a more knowledgeable peer than he/ she can alone. However, the types of assistance he referred to--demonstration, giving the first step of a solution, or asking a leading question--are more subtle than the usual techniques of direct training (Schneider, 1987; Vygotsky, 1986). The type of assistance given in the Developmental Activities Program of Phillips and Phillips is more along the lines of the type suggested by Vygotsky--questioning, starting a pattern for a child to complete, challenging a child's hypothesis, etc. However, regardless of the approach, the final goal and test is the child's ability to transfer a structure to a new situation and to solve problems on his/ her own.

(cited in Tamburrini, 1982, pp. 311-312).

Another explanation of the success of training in some cases is the notion that children just happened to be in a stage of transition when the training occurred; that they had already achieved a new way of reasoning or an increased information-processing capacity. If the child is not at a transition stage the training is ineffectual. As Carey (1987) writes:

Indeed, the hallmark of most of Piaget's phenomena is that it is nearly impossible to get the child to perform as would an adult. Upon seeing a non-conserving child for the first time, most people's initial response is that the child is failing to understand the question, for some trivial reason. The intuition is that given 20 minutes with the child, you could make clear what is wanted and straighten the kid out. The literature is littered with failed attempts to do just that. So-called training studies just do not work--unless the child is "ready" to be trained. (pp. 160-161)

Ginsburg (1988) stated frankly: "The direct teaching of Piagetian concepts--is a mistake. . . . The training program may inculcate only the surface manifestations and not the underlying structure" (pp. 244-245).

### Cross Cultural Studies

Regarding the results of cross cultural investigations, Cohen (1983) criticized Piaget's theory that children all over the world mature intellectually in the same way. He pointed out that some studies indicate, for example, that Australian Aboriginal children demonstrate a much more varied pattern in the acquisition of conservation skills than

their Western counterparts. Cross cultural studies often show third world children to be lagging behind first and second world children in their cognitive development. Cohen stated that this is often because the subjects are not familiar with testing materials and situations. He concluded that children learn to think in different ways in different cultures. Unfortunately he ignored the abundance of cross-cultural studies, including this one, that confirm Piaget's findings.

### Development of Moral Judgement

Cohen's (1983) main criticism of Piagetian research of moral judgement is the failure to account for the impact of emotions on moral reasoning. He referred to a study of juvenile delinquents who had attained high levels of moral thinking but whose behavior was askew. Their crucial choices apparently were not determined by logical thought about morals.

### Relevance to School Learning, Diagnosis, and Instruction

The contents of this section do not summarize the "fallacies" of Piaget's theory but rather point out its "limitations" in relation to education.

In search of a developmental theory of instruction which will provide a "workable basis for optimizing young children's acquisition of culturally valued skills and concepts," Case (1987) stated that

Piaget's theory provides a good starting point for building such a theory since it focuses attention on the importance of intellectual operations in the process of learning, on the fact that these operations are highly structured, on the fact that they go through a fixed sequence of reorganization in the course of development,

and on the fact that the perception of inconsistency can play a major role in motivating these reorganizations. (pp. 221-222)

Nevertheless, Case (1987) considered Piaget's theory to be insufficient for building a developmental theory of instruction because it fails to answer three important questions: (1) "how to identify the operational structures of relevance to the sorts of skill normally taught in school," (2) "how to assess children's current level of operational functioning," and (3) "how either to bring children from their current level of functioning to that which is desired, or to adapt the method of instruction such that the material in question can be mastered without a change in the current level of functioning" (p. 222).

Case presented a neo-Piagetian theory which attempts to meet these specifications. Operational structures are characterized as "sets of executive strategies" used to reach specific goals in particular problem domains. The development of operational structures consists of the reorganization of these strategies in response to either (1) the child's awareness of failures to reach task goals, or (2) the child's growing perception of the complexity of a problem area. According to Case research has confirmed the influence of four factors on the development of operational structures: (1) the size of the child's *M*-power, (2) cognitive style, (3) the perceptual organization and complexity of the experience, and (4) his/ her affective disposition. *M*-power is defined as "the maximum number of independent schemes [operational structures] that can be attended to at any moment in the absence of direct support from the perceptual field" (p. 195). *M*-power increases with age. Differences in cognitive style refers to the subject's degree of field independence vs. field dependence, i.e., the ability to overcome influences from the perceptual field or to separate an item from its context. Piagetian tasks which contain more field factors and greater complexity are more difficult for subjects who are more field dependent (as

determined by the Rod and Frame Test). Regarding the affective disposition of the learner, Case gave very little explanation and refers only to related studies. These neo-Piagetian factors are worth highlighting because they extend the four factors identified by the Piagetian model which affect cognitive development, namely, maturation, physical experience, social transmission, and equilibration.

Regarding the first insufficiency of Piaget's theory specified above, the lack of means to identify operational structures relevant to particular skills taught in school and used in various professions, specific goals are pinpointed, for example, "conserving liquid amount," then, the step-by-step process by which skilled performers arrive at the goal is carefully described. According to Case, neo-Piagetians are identifying and charting these executive strategies not only in the areas researched by Piaget such as classes, relations, mathematical logic, space, time, and causality, but also in geography, history, English composition, social science and critical thinking.

The second area of insufficiency, assessment of children's current level of functioning, is strengthened in the neo-Piagetian theory by describing the steps used by the child, erroneous or successful, in arriving at a conclusion in a problem area. This allows an instructor to pinpoint a child's error.

The last area of insufficiency, how to bring children up to a level of functioning or how to adapt the method of instruction to their level, is addressed in one of three ways:

(1) by presenting them with a task in which the inadequacy of their current strategy may be demonstrated and a model of the correct strategy may be provided; (2) by finding a qualitatively simpler executive strategy than the one normally used by adults and teaching this strategy [for example, in mathematics, learning to "count on" to find differences in situations such as making change for a purchase, instead of subtracting via regrouping using the base 10 system]; or



(3) by finding qualitatively simpler tasks so that they are solvable by means of the strategy children currently have available [for example, giving subtraction problems which do not require regrouping because it necessitates class inclusion]. (Case, 1987, p. 222)

Case developed a technology of instruction based on this theory. It is beyond the scope of this summary of Piagetian critiques to detail its components. Suffice it to say that the method entails intensive, one-on-one, mediated learning between the subject and the instructor. According to Case, he and his colleagues have had success with it. However, Case admitted that the cost of implementing the method in general education would be high and that it is more appropriate for the mentally retarded; for anyone trying to master material that is not easy to assimilate; for children from atypical or minority backgrounds whose performance lags far behind the norm; for a situation in which the alternative of delaying instruction for several years is unacceptable; and when a target task is a prerequisite for the acquisition of a culturally valued skill. For most children, Case recommended the more realistic and economical approach of enhancing the normal course of development by simply selecting "the type of tasks that children are given so that they will not need to apply more sophisticated operative routines than they already have available in their repertoires" (Case, 1987, p. 220).

It is of interest to note that the Developmental Activities Program developed by D. R. Phillips and D. G. Phillips (Phillips, et. al., 1994) accomplishes this recommendation of Case by providing activities with "multiple entry points," that is, activities that are rich enough to allow children with a wide range of cognitive levels to approach them and to be enthusiastically engaged by them. Advances are then made by the child him-/herself or through careful questioning by the teacher, questioning which suggests, challenges, or gently points out inconsistencies.

However, the Developmental Activities Program is only one of several which

claim to be Piagetian-based. Brodzinsky, Sigel, and Golinkoff (1981) pointed out that one of the faults of Piaget's constructivist model is that it provides no direct linkage with classroom applications. The complex, highly general, and ever-evolving nature of the theory leads to individual differences of interpretation and requires the educator to derive relevant principles and procedures for implementation. The authors pointed out that "this is apparent in the variation in educational programming, each of which claims to be 'Piagetian-based' (e.g., Copple, Sigel, & Saunders, 1979; Kamii, Chapter 15; Lavatelli, 1970; Smock, Chapter 4; Weikart, Rogers, Adcock, & McClelland, 1970)" (p. 17).

Ginsburg (1988), in his review of the contributions and limits of genetic epistemology to the field of education, stated:

The very nature of Piaget's theory sets strong limits on its potential contribution to education. In particular, the theory has little to say about cultural knowledge, individual differences, the social context of education, and certain modes of learning prevalent in the classroom. This of course is no criticism of Piaget's theory itself. Although it already deals with an incredibly wide range of phenomena, the theory cannot be expected to concern itself with everything. (p. 244)

Ginsburg went on to point out the aspects of education and schooling with which Piaget's theory fails to be concerned. By emphasizing the hypothetico-deductive method of scientific reasoning, the theory ignores other aspects of scientific activity such as exploration, the formation of hypotheses by analogy and intuition, the role of luck, and serendipitous discoveries. The theory does not directly account for the more passive, receptive, rote learning that is a common and legitimate aspect of school learning. The notion of equilibration is only in its formative stages as a theory and the

corollary role and contribution of cognitive conflict, although undoubtedly valuable, is unable to guide the educator in selecting which aspects of school learning would benefit from its application. The theory provides the theoretical underpinnings for self-directed learning, which Ginsburg acknowledges as a legitimate goal of education, but it does not explain why some students in some situations need to have their learning directed (and sometimes even forced upon them) by an authority figure. Piaget ignored perceptual learning and emphasizes thought, hence, his theory is unable to explain how people abstract knowledge directly from their perceptual experiences of the real world. Piaget's notion of logico-mathematical experience--the learner's reflections on his/ her own actions on the world--is extremely useful to educators but its application to particular aspects of schooling is unclear. In the realm of social learning, Piaget acknowledged the benefits of peer interaction for stimulating cognitive conflict leading to equilibration, but he does not go beyond this to address the transmission of values and cultural knowledge via social interaction. Regarding the role of teachers and adults, the theory fails to concern itself with how they might intervene to promote development and learning. Concerning academic knowledge and culturally derived thought in general, Piaget's theory does not deal with the acquisition of this symbolized and codified type of knowledge (Ginsburg, 1988, p. 244).

Hughes (1983, p. 17, p. 22) interpreted Piaget's belief that mathematics has foundations in the natural, spontaneous growth of the child's logical capabilities which cannot be transmitted to the learner by the teacher, as meaning that Piaget regards the teacher's role as being non-interventionist and basically unimportant because mathematics is essentially constructed by the children on their own.

This criticism is misdirected. Piaget was interested in how concepts develop, not how to develop concepts (Wadsworth, 1984). It is the field of education which should be criticized for its inability to affect a widescale operationalization of Piaget's findings in order to stimulate and assure the cognitive development of children. Hughes (1983),

himself, stated that "at present truly Piagetian classrooms are few and far between" (p. 18).

Fortunately, a handful of investigators and educators have found ways in which the teacher can "intervene" to enhance the child's construction of mental structures and mathematical abilities at an optimum rate. Among them figure Copeland, Devries, Elkind, Feuerstein, Jordan and Streets, Kamii, Kohlberg, Labinowicz, D. G. Phillips, D. R. Phillips, and Weikart. The idea that teachers are able to enhance cognitive development by structuring the learning environment and by guiding and facilitating students' interactions with it, especially by asking carefully chosen questions to create a moderate degree of cognitive conflict, has now been positively confirmed by the research of D. G. Phillips (1989). Hence, this type of intervention is preferable to training procedures characterized by "transmission" via demonstration, explanation, and positive reinforcement (Cohen, 1983; Gelman & Baillargeon, 1983; Ginsburg, 1988; Tamburini, 1982).

### Aims of Education

Although Piaget was not primarily concerned with applying his discoveries to the educational field, he did comment often about the educational enterprise and, no doubt, his ideas have influenced many people. One area which he addressed was the purpose of education:

The principal goal of education is to create men who are capable of doing new things, not simply repeating what other generations have done - men who are creative, inventive, and discoverers. The second goal of education is to form minds which can be critical, can verify, and not accept everything that is offered. The greater danger today is of slogans, collective opinions, ready made trends of

thought. We have to be able to resist them individually, to criticize, to distinguish between what is proven and what is not. So we need pupils who are active, who learn early to find out by themselves, partly by their own spontaneous activity and partly through material we set up for them, who learn early to tell what is verifiable and what is simply the first idea to come. (as cited in Elkind, 1989, p. 116)

This goal was summarized by Elkind (1989) as: "The aim of education . . . is to produce thinkers who are creative and critical" (p. 116).

Personally I find this to be too cognitive. The qualities he praises could also describe the political and intellectual leaders of the German Third Reich—one of the most highly educated group of leaders in history. They had a theory, a scientific research program, and a political program based on the concept of the racial superiority of the Aryan people. Missing in Piaget's goal statement is the inclusion of a reference to the universal, spiritual/ philosophical principles of the oneness of the human race and the organic unity of the entire creation; principles which have now been corroborated by all of the human and physical sciences: physics, genetics, neurology, biology, ecology, psychology, sociology, anthropology, and others.

Elsewhere, Piaget stated, "To educate is to adapt the child to an adult social environment, in other words, to change the individual's psycho-biological constitution in terms of the totality of collective realities to which the community consciously attributes a certain value [1970, p. 137]" (as cited in Ginsburg, 1988, p. 257). Again, such an aim would not prevent human atrocities committed in the name of any type of superior/ inferior concept whether it be of race, nationality, economic system, social class, religion, sex, political party, or ontological level (in the case of ecological disasters). This statement would fit any prejudice-based society which required its oncoming generations to accept and adapt to its twisted system of values. Adaptation is an amoral concept. Unless the question, "Adapt to what values?" is asked, it remains

a double-edged sword which could be used for any purposes--good or evil. And, because there is no perfect, adult social environment in this world to which a child can adapt, education should include as one of its aims the reform and transformation of society in universally beneficial directions.

My own formulation of the goal of education attempts to correct these weaknesses:

The aim of education is to enable the person to consciously and continuously: (1) discover, actualize, expand, and refine, at an optimum rate, his/ her potentialities and special, God-given talents which are physical, social, psychological, and/ or spiritual in nature; (2) acquire beneficial knowledge; (3) know and love the Creator and His/ Her creation; (4) actualize the potentiality of the world, that is, to strive to effect the highest good for all people and all things everywhere; (5) carry forward an ever-advancing civilization toward ever-wider circles of unity; and (6) prepare his/ her soul for the afterlife.

Such a goal statement is more comprehensive than the statements of Piaget. It subsumes his worthwhile goals while going beyond them to include the acquisition of cultural knowledge and universal values in the pursuit of personal and social transformation. This statement would be shunned by any political regime or social system which oppresses the members of any population, human or otherwise. For non-religious educational systems, by eliminating the religious/ spiritual terms, the statement would still avoid the omissions of Piaget that Flavell referred to above.

### What Remains after the Bathwater Has Been Thrown Out?

This summary of the many criticisms of Piaget's work leaves one wondering

whether or not he made any lasting contributions to fields such as developmental psychology, education, and social research. This final section, in order to somewhat balance out the picture, will summarize some of the components of Piaget's legacy which appear to be withstanding the test of time.

### Logical Thinking

Piaget's finding that children's thinking is qualitatively different from that of adults is monumental. The overwhelming success of replication studies which use Piagetian tasks is a lasting contribution to the efforts of adults to understand and chart the development of children's cognition in basic areas such as classes, number logic, relations, space, time, causality, and measurement (Elkind, 1988; D. G. Phillips, 1996).

Gardner (1983) stated:

The sequence of development . . . --Piaget's account of the passage from sensori-motor actions to concrete to formal operations--is the best worked-out trajectory of growth in all of developmental psychology. While many parts of it are susceptible to criticism, it remains the account of development against which all other formulations continue to be judged. . .

What I wish to stress here is that Piaget did pose the right questions and achieve the crucial insights about the main factors involved in logical-mathematical development. He shrewdly discerned the origins of logical-mathematical intelligence in the child's actions upon the physical world; the crucial importance of the discovery of number; the gradual transition from physical manipulation of objects to interiorized transformations of actions; the significance of relations among actions themselves; and the special nature of higher tiers of development, where the individual begins to work with hypothetical

statements and to explore the relationships and implications which obtain among these statements. To be sure, the realms of number, mathematics, logic, and science are not coextensive with one another. . . . But that they do form a family of interrelated competences seems to me true: one of Piaget's enduring contributions is to have suggested some of the integrating links. (pp. 133-134)

### Educational Curricula

Piaget's research confirms the notion that the child's current stage of cognitive development places limits on what he/ she can learn. Therefore, because certain curricular concepts require particular mental structures for their understanding, curricular materials need to be adjusted so that they will be in line with the child's level of intellectual development. For certain academic areas, this provides the justification for a more individualized and personalized approach to curriculum delivery (Ginsburg, 1988).

Elkind (1988) stated, "Recent studies suggest that children's success on tests and with curricula is related to the 'fit' between the child's stage presupposed by the tests and curricula. This means that educational practice can be improved by a better match between the child's level of development and that imbedded in the tests and curricula" (pp. 97-98).

### Learning Theory

Piaget's work provided theoretical support for active approaches to learning; "active" in the sense of structuring learning experiences which involve both physical and mental engagement and commitment. His theory that the child plays a vital role in directing the course of cognitive development which cannot be transmitted by direct



instruction, also lends support for extensive self-directed learning in at least some academic situations (Ginsburg, 1988).

### Bodily-Kinesthetic/ Sensori-Motor Intelligence

Bodily-kinesthetic intelligence according to Gardner refers to

the capacity to use your whole body or parts of your body--your hand, your fingers, your arms--to solve a problem, make something, or put on some kind of a production. The most evident examples are people in athletics or the performing arts, particularly dance or acting (as cited by Checkley, 1997, p. 12).

Although Piaget was not concerned with bodily intelligence, Gardner (1983) credits him with having made a significant contribution: "His [Piaget's] description of the unfolding of sensori-motor intelligence, in fact, illuminates its [bodily-kinesthetic intelligence's] initial evolution" (p. 220).

### Logico-Mathematical Experience

Ginsburg (1988) considered the notion of logico-mathematical experience--reflecting on one's actions with the physical world--to be unique, very valuable, and extremely useful for teachers, even though insight is still lacking as to how to apply it to the educational setting.

### Spatial Intelligence

Spatial intelligence according to Gardner

refers to the ability to represent the spatial world internally in your mind--the way a sailor or airplane pilot navigates the large spatial world, or the way a chess player or sculptor represents a more circumscribed spatial world. Spatial intelligence can be used in the arts or in the sciences. If you are spatially intelligent and oriented toward the arts, you are more likely to become a painter or a sculptor or an architect than say, a musician or a writer. Similarly, certain sciences like anatomy or topology emphasize spatial intelligence" (as cited by Checkley, 1997, p. 12).

This is still another one of the "multiple intelligences" to which Piaget has made a particular contribution. Gardner (1983) stated:

Though the centrality of spatial intelligence has long been recognized by researchers who work with adult subjects, relatively little has been definitively established about the development of this set of capacities in children. . . .

An exception is Jean Piaget, who conducted several studies of the development of spatial understanding in children. (p. 178)

Gardner goes on to discuss some of Piaget's investigations. I think Gardner would be delighted with the D. G. Phillips' "Sciencing" program which does so much to educate spatial intelligence.

### Language

Although, as discussed above, Piaget perhaps underestimated the important role of language in development, Ginsburg (1988) credited him with having abolished the

myth that children simply learn by listening. Besides helping educators to understand that they must first assess the level of understanding of the child before giving verbal explanations, Piaget showed that each child's language has a distinctive meaning for the child--regardless of the dictionary meaning of the words used.

### Peer Interaction

As mentioned previously, Piaget's work supports the use of teaching methodologies which incorporate peer interaction. Debates, discussions, forums, and panel discussions facilitate the exchange of ideas and intellectual conflict which stimulate disequilibrium and cognitive growth (Ginsburg, 1988).

### Children's Intuitive Knowledge

Ginsburg (1988) commended Piagetian theory for its optimistic portrayal of the spontaneously developed thought structures which can be used to assimilate academic knowledge. He agreed with and quoted Piaget who stated that,

The pedagogic problem . . . still subsists in its entirety: that of finding the most adequate methods for bridging the transition between these natural but nonreflective structures [that is, the child's spontaneously developed intuition which have not entered conscious awareness] to conscious reflection upon such structures and to a theoretical formulation of them [1970, p. 47] (as cited in Ginsburg, 1988, p. 254).

This concern with understanding and guiding children's transition from intuitive knowledge to academic knowledge appears to be the focus of the latter part of

Ginsburg's career. In *Piaget's Theory of Intellectual Development* which he wrote with Sylvia Oppen (first published in 1969) he described several Piagetian concepts which have direct relevance to education. He acknowledged, however, that, as a psychologist, he did not have the skills needed to implement them: "Like Piaget, we feel that the implementation of these principles requires the special skills of the educator, who understands the distinctive conditions of the school setting, rather than the psychologist."

In his article "Piaget and Education: the Contributions and Limits of Genetic Epistemology" (which was, according to my investigations, first published in 1981) he pointed out the contribution which he believed psychologists and other investigators could make to addressing the central concern of pedagogy:

At the very least we can say that it is not clear that there is a strong relation between the Piagetian structures and the kinds of thought processes involved in school learning. To a large extent the question is an empirical one, since we have very little knowledge concerning the thought processes actually employed in academic learning. A productive approach, I think, is for those with a Piagetian orientation to undertake direct investigation of academic cognition in order to determine whether the Piagetian notions are indeed useful, or whether new accounts need to be developed. (257-258)

Ginsburg, apparently, was giving this advice based on his own experiences with the investigations of arithmetic structures which he delineated in *Children's Arithmetic* which was first published in 1977 and then updated in a 1989 edition.

In the light of these concerns and comments I think that the contributions of Darrell G. Phillips and Dale R. Phillips (1989, 1991, 1994, 1996) are historical. As experienced educators and professional researchers thoroughly grounded in Piagetian

theory, they have bridged many of the implementation gaps so sorely bemoaned by psychologists and educators. Their research has found a direct link between the Piagetian mental structures and the academic areas of math and science, namely, those aspects which require logical thinking. In other words, they have found a niche in the educational curriculum where Piaget's work can be applied. They have found that the development of structures is not just spontaneous but can be stimulated and enhanced through the appropriate intervention of trained adults. They have developed an individualized, elementary math program which introduces concepts when the child shows that he/ she has the necessary structures for understanding them. And they have created an instructional system which incorporates classroom management techniques, guidelines for selecting materials, questioning techniques for teachers, an assessment program, a record keeping systems, and much more.

Personally, I believe that Ginsburg, along with many of the other critics cited in this summary, would be delighted with this breakthrough.

### Equilibration

Fincham (1982) pointed out that

possibly the most important [of Piaget's fundamental concepts] is equilibration. The idea of an internal mechanism of self-regulation co-ordinating changes in maturation, physical and social experience seems valuable. . . . The active equilibrium proposed by Piaget serves to remind us of the continually changing nature of the system and points to the danger of remediation based on a single, one-shot diagnosis, (p. 384)

Gelman and Baillargeon (1983) confessed that "we still are far from a full

understanding of the various processes postulated in Piaget's treatment of reflective abstraction and equilibration." They emphasized, however, that

there can be no denying something like assimilation and accommodation as being involved in learning and development. . . . Whether Piaget's particular version of how schemes develop will stand the test of time, we do not know. But we are sure that notions akin to assimilation and accommodation will. And by now, they are no more mysterious to us than are the processes of association and selective attention. (p. 217)

### Cognitive Conflict

Brodzinsky, Sigel, and Golinkoff (1981) in their review of papers by Gallagher, Reid, Forman, Kuhn, and Duckworth, acknowledged that although Piaget's theory fails to clearly explain why "contradictions"--discrepancies between the person's mental structure and an anomalous event he/ she encounters--stimulate the equilibration process, there is general consensus that "to help children become active problem solvers and active thinkers, employment of contradictions and surprises may well be advocated, even though the mechanics of the process are not well understood" (p. 21).

Ginsburg (1988) stated, "The strategy of deliberately jarring the student's cognitive structure and thereby enhancing active learning is an important idea for education" (p. 320).

### Clinical Method

Fincham (1982) stated that Piaget's clinical method is "undoubtedly one of the major factors that made possible his contribution to knowledge" because it not only

revealed many new facts but it also "drew attention to the distinction between the product and processes of thought" (p. 383). Fincham further commented that "because it [the clinical method] stresses how a child functions rather than the level at which he performs, it has implications for differential diagnosis which when conceived in these terms leads more directly to remediation" (pp. 383-384).

Vygotsky (1986) stated that Piaget's "clinical method proves a truly invaluable tool for studying the complex structural wholes of the child's thought in its evolutionary transformations. It unifies his diverse investigations and gives us coherent, detailed, real-life pictures of the child's thinking" (p. 14).

Ginsburg (1988) highly recommended the clinical method as an individualized, flexible method of testing, but he admonished educators to extend its use beyond Piagetian content. He recommended that educators use it to "observe, via the richness of the clinical method, the unusual patterns of reasoning displayed by young children as they grapple with ordinary school arithmetic" (p. 247).

### Conclusion

At the end of one's career, it is better to be prepared to change one's perspective than to be condemned to repeat oneself indefinitely.

--Jean Piaget (as cited in Inhelder, 1982)

No scientific theory is final and even Piaget, credited by Howard Gardner as being "the single dominant thinker in his field" (1991, p. 28), has referred to himself as his most adamant critic. Theoretical change will continue as a dialectical process in the sense that Piagetian researchers will continue to respond to criticism from within their school and from without. Revisions, no doubt, are needed but the challenge, in my opinion, is to throw out the bathwater without dumping out the baby. As M. Boden, the

author of *Piaget* stated, "In sum, despite all the criticisms, there is a rich store of psychological insights and theoretical speculations, and a profusion of intriguing empirical observations and remarkably ingenious experiments, to be found in Piaget's pioneering work" (as cited in Modgil and Modgil, 1982, p. 8). As efforts are made by psychologists and educators to rethink, build upon, and eventually transcend Piaget's work by creating a "new comprehensive theory of child development" (Cohen, 1983, p. 99), the counsel of Siegel and Brainerd, authors of *Alternatives to Piaget*, seems most appropriate: "The more prudent course lies somewhere between totally rejecting the theory and refusing to recognize its weaknesses" (as cited in Modgil and Modgil, 1982, p. 8).

I came to this research with the purpose of enriching the Wholistic Educational System which is being created by myself and a group of educators and scholars in various fields. My search has been well-rewarded. In the Piagetian model I have found an extensively researched approach to the development of cognitive structures underlying logic, mathematics, and aspects of science such as space, time, and measurement. The Piagetian theory of cognitive development is grounded in principles of organism and process; principles which are in harmony with my religious beliefs and my philosophical perspective on reality and life. In the Developmental Activities Program of D. G. Phillips and D. R. Phillips (Phillips, et. al., 1994) I have found a practical, research-based program with a curriculum for developing logical thinking, basic math competencies, and science process skills.

What I was not looking for and did not find were comprehensive theories of human development, education, and learning. As this summary of criticisms amply attests, those who have sought these in Piaget have found only disappointment; a disappointment which is surprising because it was never Piaget's purpose to solve the problems facing the field of education. For a more comprehensive set of educational principles and ideals I have drawn upon the writings of the Baha'i Faith on education



(Bahá'u'lláh, 'Abdu'l-Baha, & Shoghi Effendi, 1987) and, for a thoroughly researched set of comprehensive theories, I have utilized the Anisa Model of Education (Jordan & Streets, 1973a). My endeavors to integrate the best thinking and research in the fields of religion, philosophy, science, and education into a comprehensive system of education continue.

In conclusion, as an educator interested in making a significant contribution to the construction of a new global order based on concepts of unity, social justice, and peace, I support wholeheartedly the recommendation of Siegel and Hodkin (1982b):

So, instead of fighting each other's personal paradigms of cognitive development, I would propose that psychologists of all schools unite in their efforts to secure these minimum conditions of life-long development [health and wealth] for the children of the world especially in these times when so many children are abused, tortured, starved or killed. (p. 99)

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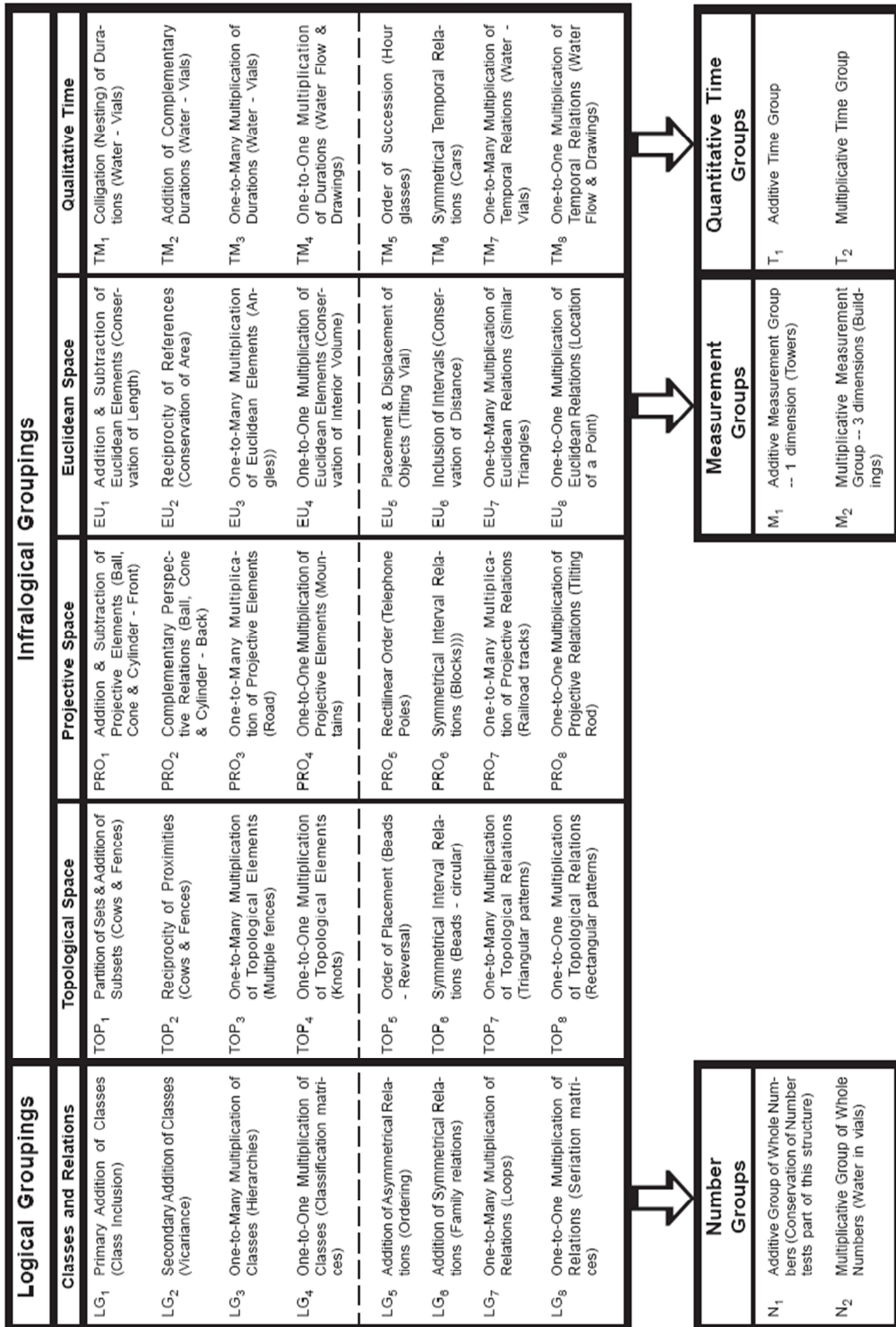
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APPENDIX A:  
CHART OF CONCRETE OPERATIONAL STRUCTURES





APPENDIX B:  
EQUIPMENT, PROCEDURE, SCORING CRITERIA, AND SCORING SHEETS  
FOR TASK 1--COLLECTIONS

**INTERVIEW PROTOCOL**  
**for**  
**Graphic & Non-Graphic Collections**  
**(Paper Shapes)**

**Equipment**

(Note: This task may be given with many different types of objects and different numbers of objects. The particular objects used should lend themselves to ease of manipulation and should certainly allow for all of the various categories of graphic and non-graphic collections.)

The equipment consists of 37 paper or tag board shapes as described below:

Squares:

Large: 8 cm per side

Small: 5 cm per side

Rectangles:

Large: 3 cm  $\times$  7 cm

Small: 1.5 cm  $\times$  4.5 cm

Right triangles:

Large: 8.5 cm base, 4.25 cm height

Small: 5.7 cm base, 2.85 cm height

Isosceles triangles:

4 cm base, 8 cm height

Half-rings:

4 cm outside radius, 2 cm inside radius

Number of Paper Pieces of Each Color:

Large Squares: 2 red, 2 yellow, 2 blue

Small Squares: 2 green, 2 blue

Large Right Triangles: 1 red, 1 yellow

Small Right Triangles: 2 green, 2 blue

Large Rectangles: 1 red, 2 yellow, 2 green, 2 blue

Small Rectangles: 4 red

Half-Rings: 2 each of red, yellow, green and blue

Isosceles Triangles: 1 yellow, 1 blue

(Note: A master for these shapes is included in Appendix B.)

**Preparation**

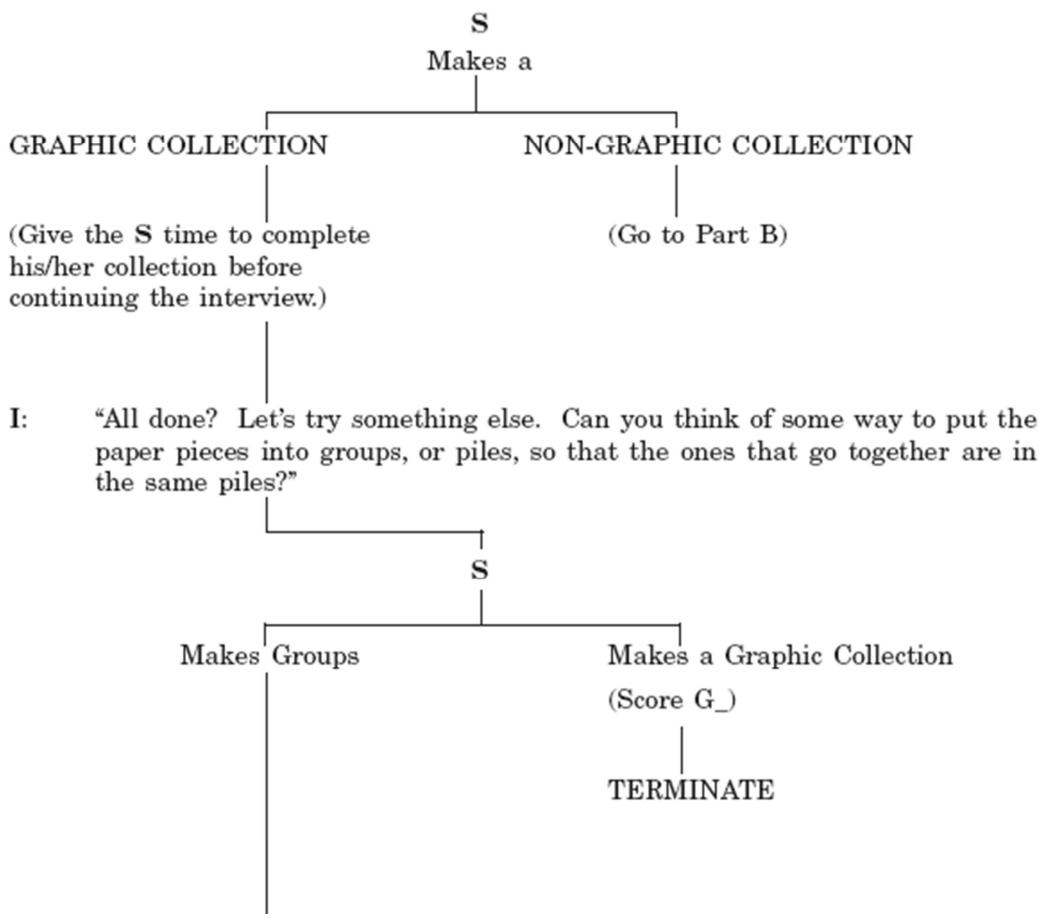
The shapes should be well mixed.

**Procedure**

There may be some difficulty in communicating to young children just what is expected of them. The opening statement that is suggested below can be interpreted as meaning, "Make a design." Even so, the opening must be totally noncommittal; it does little good to conduct an interview if the child is cued or led in any way.

PART A — Initial presentation of all shapes

- I: “Here I have some pieces of paper. Look at them carefully. I would like for you to put together the ones that you think should go together.”



PART B — NON-GRAPHIC COLLECTIONS

- I: “Can you show me which ones go together? Use your finger and draw a pretend line around one of your groups.” (Pause) “What is the name of that group?” (The I should ask the S to draw around and name several of the groups.)

Note: The I must now determine whether the S’s arrangement is Non-Graphic 1, 2 or 3. Some typical responses are given below.

For Non-Graphic 1:

Multiplicity of criteria (e.g., shape & color) and overlapping criteria (e.g., “squares” as a group name, but one or more squares are in another group).

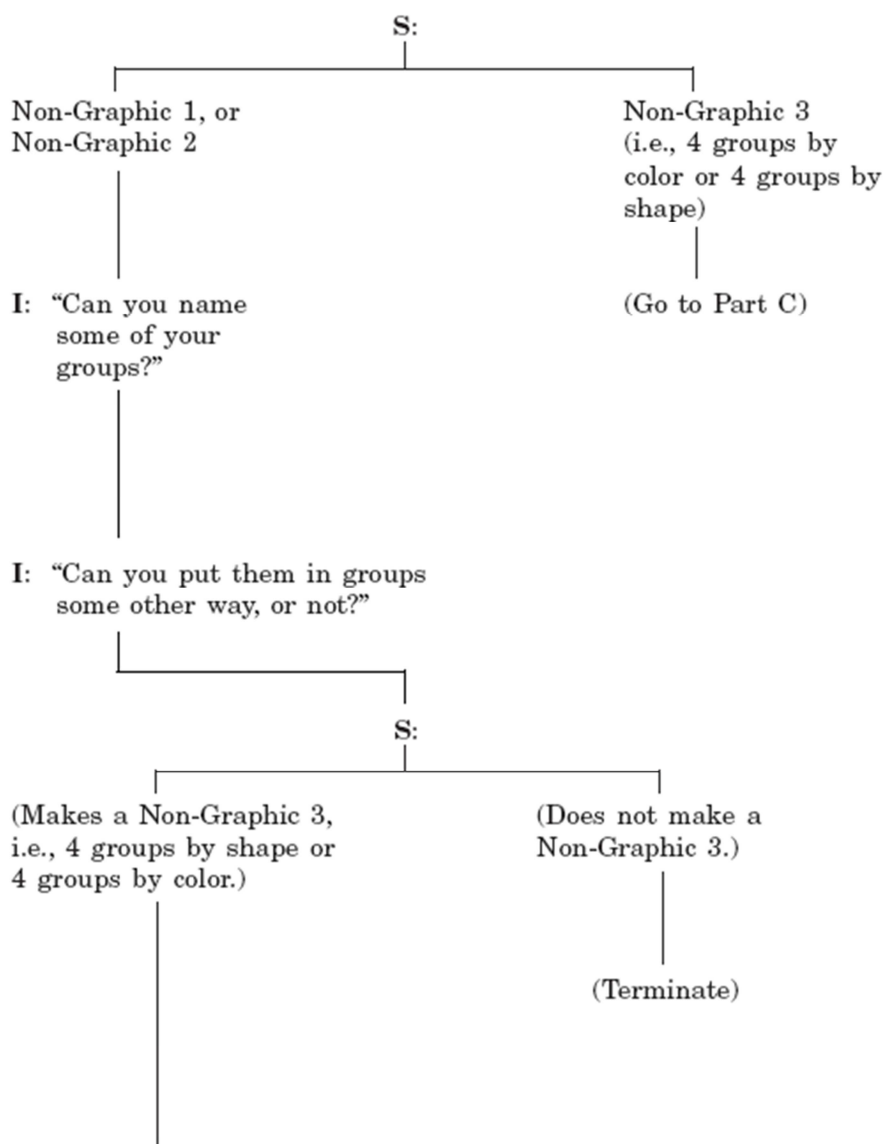
For Non-Graphic 2:

Multiplicity of criteria, but no overlapping criteria. Many groups with each thing being in its own separate group.

For Non-Graphic 3:

Four groups by color, or four groups by shape.

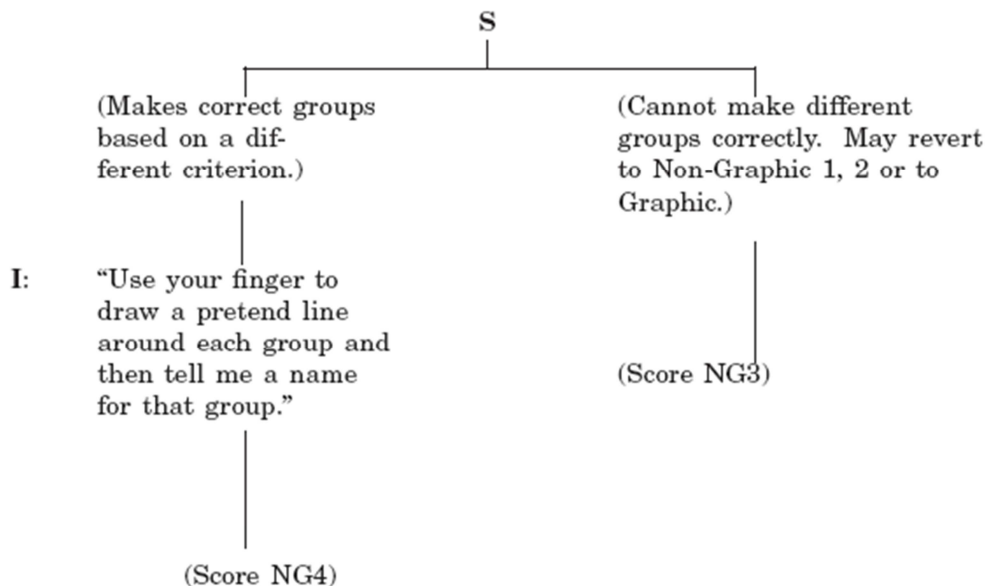
If the S has made a Non-Graphic Collection the I proceeds as follows:



#### PART C — Determination of Non-Graphic 4

(Note: The S must have made one clean arrangement, i.e., a Non-Graphic 3, or this task part cannot be given.)

I: "Can you group them in a different way, or not?"



---

### Response Categories

#### GRAPHIC COLLECTIONS

A graphic collection is first and foremost a spatially dependent array or design; the actual position of the objects is not only important, it is a necessity. The child will certainly place some of the paper pieces by matching a shape or color, but this is done in the sense of "as we go along," or "this one goes next to that one." These responses show very clearly that the child uses proximity, separation and order of placement (i.e., some of the characteristics of Topological Space) to construct his/her arrangement.

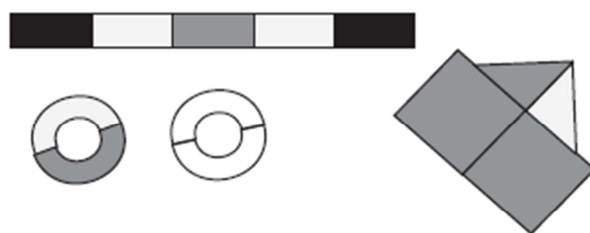
The child at the level of Graphic Collections is incapable of anticipating a classification scheme; therefore, there is no overall plan of action that accounts for and includes all the paper pieces in the set. Putting sets of objects together by means of an abstracted criterion has no meaning to the child at this level.

The categories below describe various Graphic Collection arrangements. Category 1 (Small Partial Alignments) is typically the first response observed if the child can do anything at all with the objects. The remaining six categories are not hierarchical since a child may in one instance "feel like" making a long Continuous Alignment and in the next instance make a Complex Object.





- G4 Intermediate Responses.** These fall between Alignments or Segments and Collective or Complex Objects. The **S** in this category will construct some one-dimensional arrangements (G1s, G2s, and/or G3s), but will also construct some two-dimensional arrangements (G5s and/or G6s). See the example below.

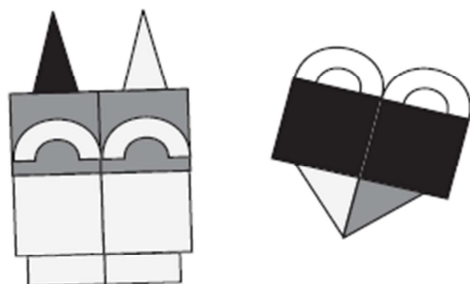


- G5 Collective Objects.** A collective object is a two- or three-dimensional collection of homogeneous (i.e., same shape) objects which together form a unified figure. (It is the use of more than one dimension that sets Categories 5 and 6 apart from categories 1, 2 and 3.) Collective objects frequently depict a geometrical shape, e.g., a group of squares placed together to form a large square, or they may be constructed in three-dimensions (i.e., stacked). Collective objects are somewhat unstable since the **S** often proceeds to add different shapes to make a design or figure. See the example below.

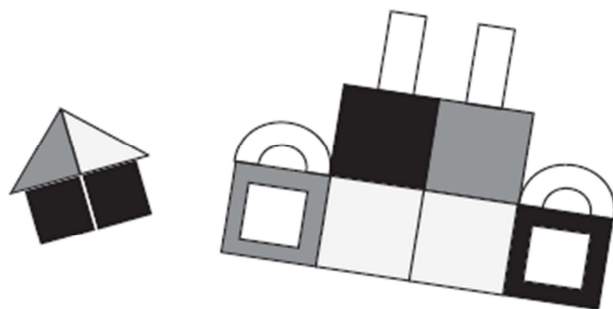


- G6a Complex Objects.** These are based upon geometrical content, that is, heterogeneous objects (different shapes) are arranged in a two- or three-dimensional geometrical pattern. Part-whole relations are reinforced because the whole becomes a closed set. The **S** tends to be fascinated by the shape of the whole and forgets about the internal relations of similarity and difference among its elements. Part-whole relations are primary while those of similarity and difference are of secondary importance. See the example below.





- G6b**     **Complex objects based on situational content.**     These complex objects have a descriptive meaning. Responses are often difficult to separate from those in the preceding category, but here one finds that the child has attached a descriptive meaning to the arrangement. For example, the child may “name” his/her arrangement as a house, factory, rocket ship, person, etc.



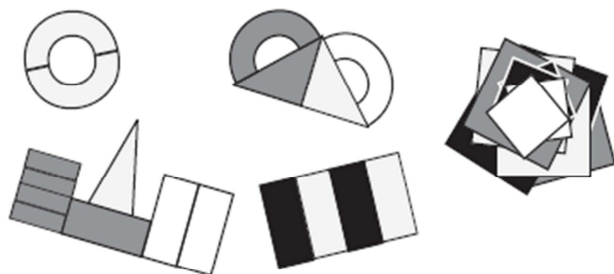
See the example below.

(Note: If the **S** does not volunteer a name for his/her arrangement the **I** may say, “Tell me about what you’ve done.” The **I** should certainly *not* ask, “Does that have a name?”)

#### MIXTURES OF GRAPHIC AND NON-GRAPHIC COLLECTIONS

The child gradually moves from graphic collections to non-graphic collections by a series of transitional steps in which there are numerous intermediate responses. Examples would be a segmented collection which is still partially graphic in nature, or the substitution of small non-graphic collections for collective or complex objects.

<u>Category</u>	<u>Description</u>
<b>GN</b>	<b>Mixture of Graphic and Non-Graphic.</b> A common example of this category is the child who constructs one or more Graphic Collections (G1 through G6) and one or more Non-Graphic Collections. In the example below the child has constructed a Non-Graphic pile of “Squares,” but has also constructed some Graphic arrangements (G1, G3, and G6). The child in this category



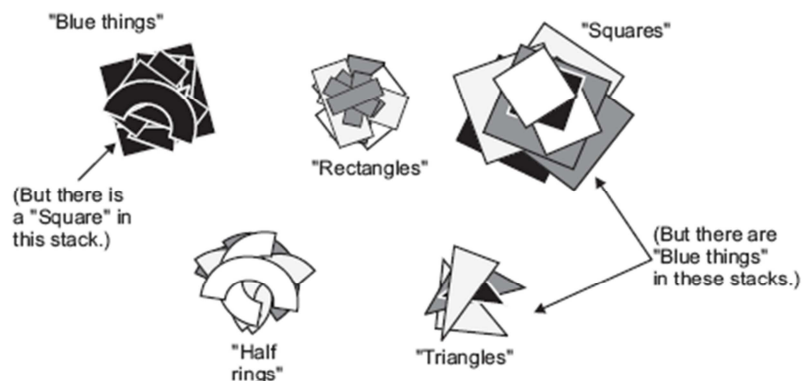
may well be thought of as “transitional.”

### NON-GRAPHIC COLLECTIONS

These collections differ from Graphic Collections in that the objects are assigned to one group or pile on the basis of similarity. Non-Graphic Collections are subordinate to the principle of spatial proximity (i.e., the objects must all be present); therefore, Non-Graphic Collections do not constitute a fully organized, hierarchical, class structure that includes non-present classes or class members.

Unlike the categories of Graphic Collections the categories of Non-Graphic Collections are hierarchical. For example, it is impossible for the S to perform at level NG3 (see below) until he/she has been through the prerequisite levels of categories NG1 and NG2.

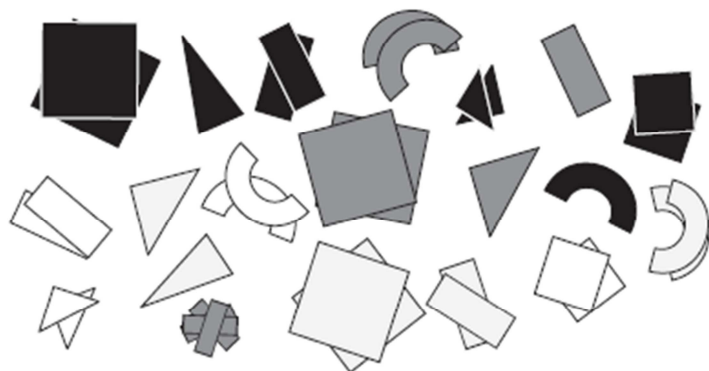
<u>Category</u>	<u>Description</u>
NG1	Several small collections based on different and overlapping criteria. Some of the objects may not be included in the final arrangement.  Overlapping criteria means that the criteria are not exclusive. In the example below, the S made groups of “squares,” “triangles,” “rectangles,” “half-rings,” and a group of “blue things.” First, the groups are based on different



criteria (i.e., shape and color), but there is also overlap as shown.

If there is to be a group of “Blue things” then *all* of the blue objects *must* be in that group. If they are not, there are overlapping criteria. The criteria are, in effect, ambiguous, or not clean.

- NG2** This category is typically a number of small collections based on a multiplicity of criteria, but without overlapping criteria and without remainder (i.e., the S uses all the paper shapes). A common example is the S who puts everything into its own group. This results in many different groups with titles such as large blue squares, small blue squares, small yellow squares, large



red triangle, small blue triangles, and on and on. (See diagram below.)

- NG3** The S in this category is able to produce one acceptable classification scheme based upon a single criterion (i.e., either color or shape with this set of objects) and there is no overlap. This results in four groups by color (see diagram below), or four groups by shape. The child at NG3 can make **one** acceptable classification scheme, but not two. The child will *either* make four



groups by color (as shown below),

or the child will make four groups by shape (as shown below).



**NG4**     The **S** in this category begins as in NG3 above (i.e., four groups by color *or* four groups by shape), and when asked if there is any other way to group the objects responds by easily changing criteria and reclassifying. For example, the **S** might initially group by color as above; then when asked for another way the **S** would regroup by shape. In both instances the classification criteria are clean, simple, few and non-overlapping. In effect, the child at NG4 can make **two** acceptable classification schemes.

**RESPONSE SHEET**  
**for**  
**Collections Task**  
**(Paper Shapes)**

Interviewer: \_\_\_\_\_ Date: \_\_\_\_\_.

Subject I.D.: \_\_\_\_\_ Age: \_\_\_\_\_ yrs. \_\_\_\_\_ mons.

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(Describe or draw S's first arrangement.)

This arrangement was a: G1 G2 G3 G4 G5 G6a G6b GN  
or an: NG1 NG2 NG3

---

(Describe or draw S's second arrangement.)

This arrangement was a: G1 G2 G3 G4 G5 G6a G6b GN  
or an: NG1 NG2 NG3 NG4

APPENDIX C:  
EQUIPMENT, PROCEDURE, SCORING CRITERIA, AND SCORING SHEETS  
FOR TASK 2--CLASS INCLUSION

INTERVIEW PROTOCOL  
for  
Primary Addition of Classes ( $LG_1$ ) — Class Inclusion  
(Animals)

**Equipment**

Toy animals — 9 cows (all alike) and 4 pigs (all alike)

**Procedure**

**PART A — Identification of Animals**

**I:** “Here I have some toy animals. Can you tell me what they are?”

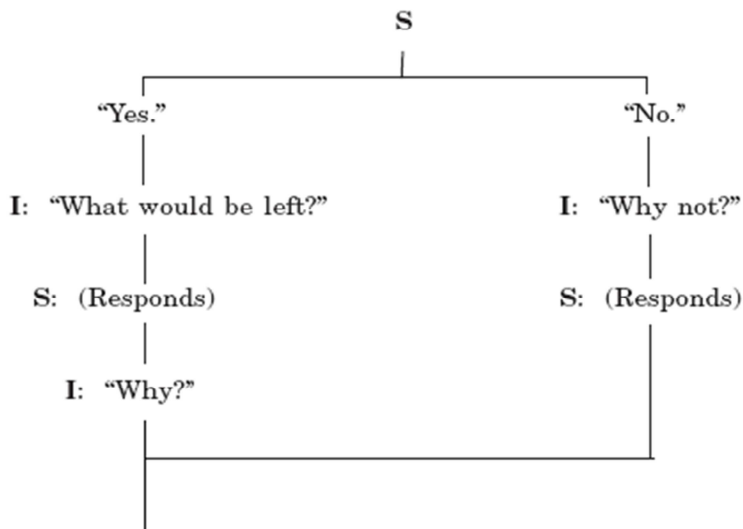
Note: The **I** should encourage the **S** to touch and pick up the animals; the **I** may help the **S** name the animals if the **S** does not recognize them or asks what one of the animals is called. The **I** must be *very sure* that the **S** can distinguish the animals from one another; if not the task should be terminated. In addition, if the **S** calls one of the animals by some other name, e.g., calls the pigs “sheep” the **I** should not correct this. The **I** should use the “sheep” label throughout the task. Tasks are not intended to be “naming activities,” nor are they teaching activities.

**I:** “Can you take your finger and draw a circle around all of the cows and just the cows? How about the pigs? Can you draw a circle around all of the pigs and just the pigs? Now use your finger and draw a circle around all of the animals.” (The **S** should do the drawing, *not* the **I**.)

Note: If the **S** cannot draw the three circles correctly the task should be terminated.

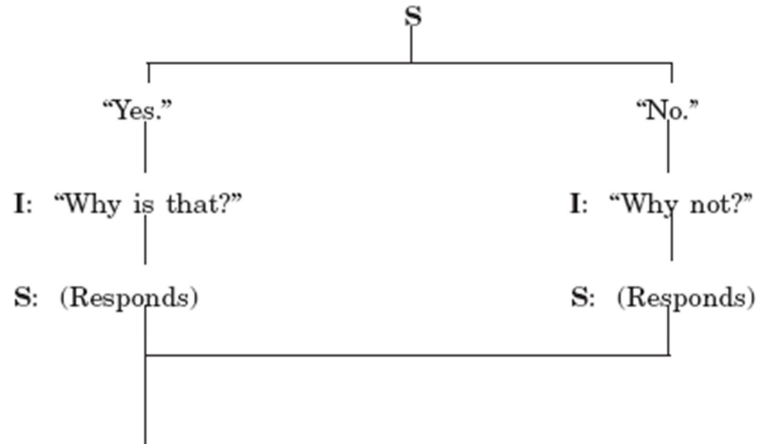
**PART B — Intensive Exclusion**

**I:** “Tell me, if we take all the cows away will there be any animals left, or not?”



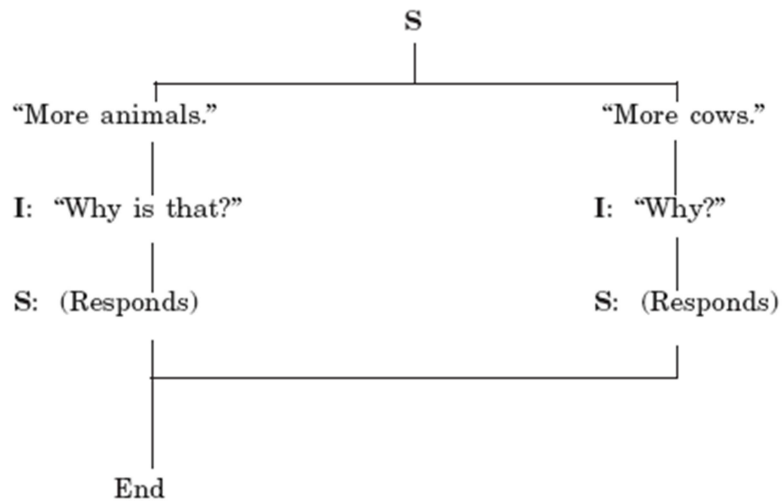
PART C — Intensive Quantification

I: “If we take all the animals away will there be any cows left, or not?”



PART D — Extensive Quantification

I: “Tell me, are there more cows or more animals?”





## Response Categories

Category	Description
0	The S does not understand that cows and pigs are animals, or the S cannot draw around the cows, pigs and animals. Part A is incorrect. Task terminated.
1	Part A correct. Parts B, C and D (choices and/or reasons) are incorrect.
2	Part A correct. Part B (choice and reason) is correct. Parts C and D (choices and/or reasons) are incorrect.
3	Part A correct. Part C (choice and reason) is correct. Parts B and D (choices and/or reasons) are incorrect.
4	Part A correct. Part D (choice and reason) is correct. Parts B and C (choices and/or reasons) are incorrect.
5	Part A correct. Parts B and C (choices and reasons) are correct. Part D (choice and/or reason) is incorrect.
6	Part A correct. Parts B and D (choices and reasons) are correct. Part C (choice and/or reason) is incorrect.
7	Part A correct. Parts C and D (choices and reasons) are correct. Part B (choice and/or reason) is incorrect.
8	All task Parts correct (choices and reasons).

## Pursuit of Reasons

If the S states a correct choice it is of utmost importance that the I pursue any insufficient reasons, particularly in Part D of this task. The S has a 50% chance of guessing a correct choice, so the I must be very sure that the S is not guessing by pursuing the reasons for any correct choices.

In Part C ("If we take all the animals away will there be any cows left or not?"), the reason, after a correct choice, should demonstrate the S's grasp of the fact that the cows are part of the group of animals. Just restating the choice, "If you took them away, the cows would be gone," is not sufficient. A typical correct reason would be, "If you took all the animals away the cows would be gone because they are animals too."

In Part D ("Are there more cows or more animals?"), the S must demonstrate by the reason that he/she understands that the cows are *included* within and therefore smaller than the class of animals. Reasons such as, "Cows are animals," or the naming of other animals (e.g., "Because there's dogs, and cats, and lions, and elephants, and . . .") are *not* sufficient and must be pursued. Such responses do not demonstrate inclusion-type reasoning.

Correct reasons must demonstrate the inclusion aspect of the two classes. Statements such as, "Well, all of these are animals and the cows are just some of them," or, "Cows are just part of the animals," are sufficient.

**RESPONSE SHEET**  
**for**  
**Class Inclusion Task (Cows & Pigs)**

Interviewer: \_\_\_\_\_ Date: \_\_\_\_\_.  
Subject I.D.: \_\_\_\_\_ Age: \_\_\_\_\_ yrs. \_\_\_\_\_ mons.

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**PART A**

1. S recognizes the animals: Yes \_\_\_ No \_\_\_ (Terminate)
  2. S draws lines around correctly: Yes \_\_\_ No \_\_\_ (Terminate)
- 

**PART B**

- I: "If we take all the cows away, any animals left, or not?"  
S: "Yes." \_\_\_\_\_ S: "No." \_\_\_\_\_  
I: "What would be left?" \_\_\_\_\_ I: "Why not?"  
S: Reason:
- 

**PART C**

- I: "If take all animals away, any cows left, or not?"  
S: "Yes." \_\_\_\_\_ S: "No." \_\_\_\_\_  
I: "Why is that?" \_\_\_\_\_ I: "Why not?"  
S: Reason:
- 

**PART D**

- I: "Are there more cows or more animals?"  
S: "More animals?" \_\_\_\_\_ S: "More cows." \_\_\_\_\_  
I: "Why do you think so?" \_\_\_\_\_ I: "Why you think so?"  
S: Reason:
- 

Scoring Category \_\_\_\_\_

APPENDIX D:  
EQUIPMENT, PROCEDURE, SCORING CRITERIA, AND SCORING SHEETS  
FOR TASK 3--ORDERING BY LENGTH

**INTERVIEW PROTOCOL**  
for  
**Addition of Asymmetrical Relations (LG<sub>5</sub>)**  
(Ordering by Length)

**Equipment**

1. 12 dowels or straws.

The dowels should be identical in all respects (i.e., color, diameter, etc.) except length. They are cut to the lengths shown below (in centimeters):

Set A

A.	16.0	F.	11.0
B.	15.0	G.	10.0
C.	14.0	H.	9.0
D.	13.0	I.	8.0
E.	12.0		

Set B

b.	14.5
e.	11.5
g.	9.5

2. 12 posterboard circles.

Diameters (in centimeters) and colors:

A.	15.0 - red	e.	9.0 - yellow
B.	14.0 - green	F.	8.0 - blue
b.	13.0 - yellow	G.	7.0 - red
C.	12.0 - blue	g.	6.0 - blue
D.	11.0 - green	H.	5.0 - green
E.	10.0 - red	I.	4.0 - yellow

(Note: A master for these circles is included in Appendix B.)

**Preparation**

1. The dowels must be separated into two sets:

Set A: Dowels A, B, C, D, E, F, G, H, and I

Set B: Dowels b, e, and g

2. The posterboard circles must be well-mixed.

Note: It is important that the S *not* see all of the task equipment at the outset. The S should see the rods of Set B *only* if he/she has ordered the rods from Set A correctly. The S should see the circles *only* if he/she has ordered Set A and inserted Set B correctly.

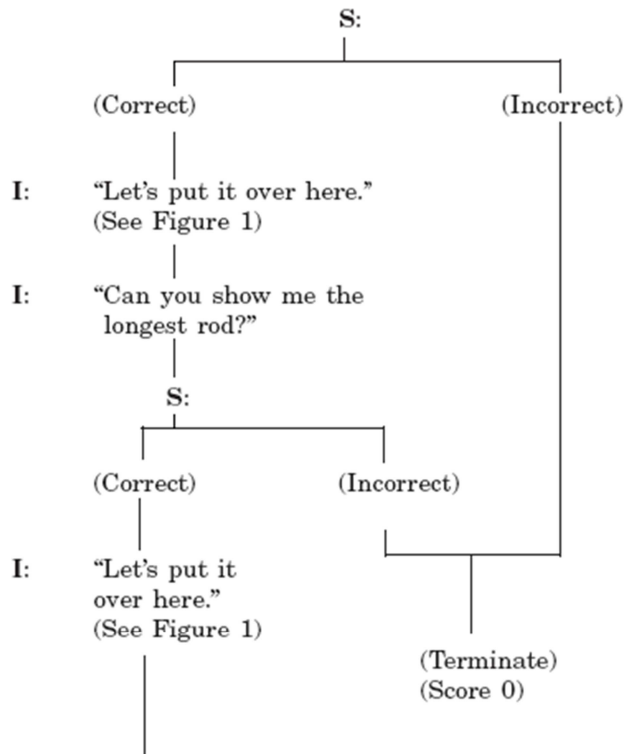
## Procedure

### PART A — Ordering Set A

- I: “Here I have some wooden rods (or straws). I would like for you to put them in order from the shortest one to the longest one.” (The I gives Set A to the S.) “Do you know what I mean when I say put them in order?”

Note: Some Ss will not know the meaning of “in order.” If so the I must attempt to communicate the meaning without providing cues or leading. The I should try statements such as, “Place the rods side by side so that they make something like stair steps.” If all else fails, try the following:

- I: “Can you show me the shortest rod?”

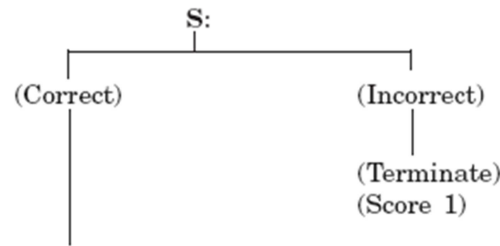


- I: “Now can you put the other rods in between these two so that they make stair steps, so they get longer and longer?” (The I indicates from the S’s left to right.)

(Note: The I must not suggest the idea of a straight base line for the set of rods. The S must do this on his/her own.)

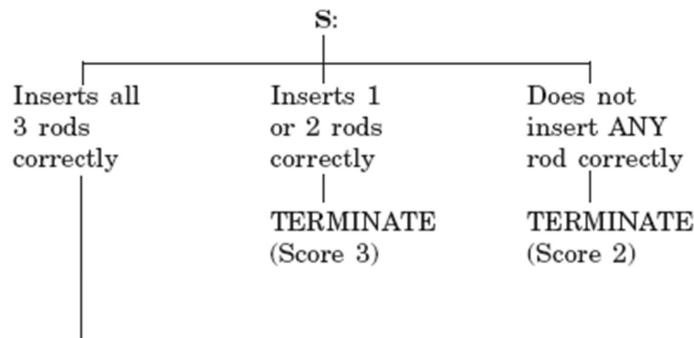


Figure 1



PART B — Inserting into the existing set

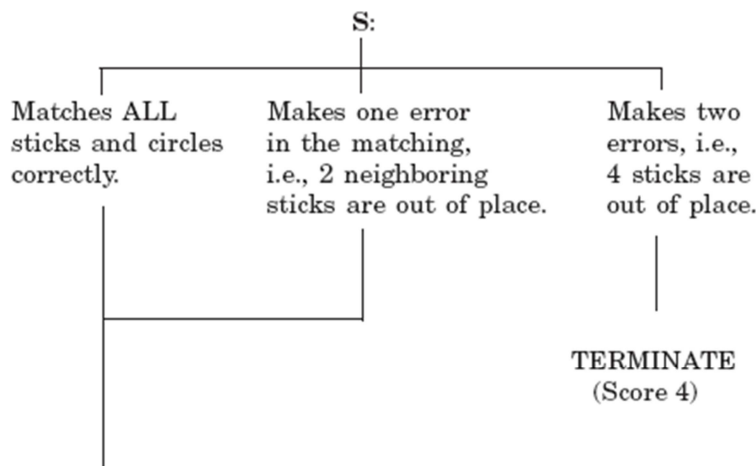
- I: "Here I have another rod. Can you put it into the set where it belongs?"  
(The I gives the S a rod selected at random from Set B.)
- S: (Responds)
- I: "And here is another rod. Can you put it into the set where it belongs?"  
(The I presents a 2nd rod from Set B.)
- S: (Responds)
- I: "And here's one more rod. Can you put it where it belongs?" (The I presents the final rod from Set B.)



PART C — Ordinal correspondence between two sets

- I: "Let's pretend that you work in a lollipop factory and that your job is to match the right sticks with the right lollipops. Here are some pretend lollipops. (The I spreads the circles out on the table.) Make your arrangement here on the table so that the biggest lollipop is with the longest stick and

the next biggest lollipop is with the next longest stick and so on all the way down until the smallest lollipop is with the shortest stick. You may move the sticks and circles any way you like.” (Note: The **I** must *never* suggest that the circles be ordered. The **S** must do this on his/her own. In addition, do not allow the **S** to stack or overlap the circles. If this happens, the **I** should say, “Let’s spread these out so you can see them better.”)



#### PART D — Maintaining correspondence between two ordered sets

- I:** “Now put the rods back like they were before.” (The **I** must be sure that the **S** puts the rods back into an ordered set. If the **S** does not put all 12 rods back correctly the **I** may say, “Now are all of the sticks in order from the shortest to the longest?” If the **S** replies, “Yes,” the task must be terminated.)

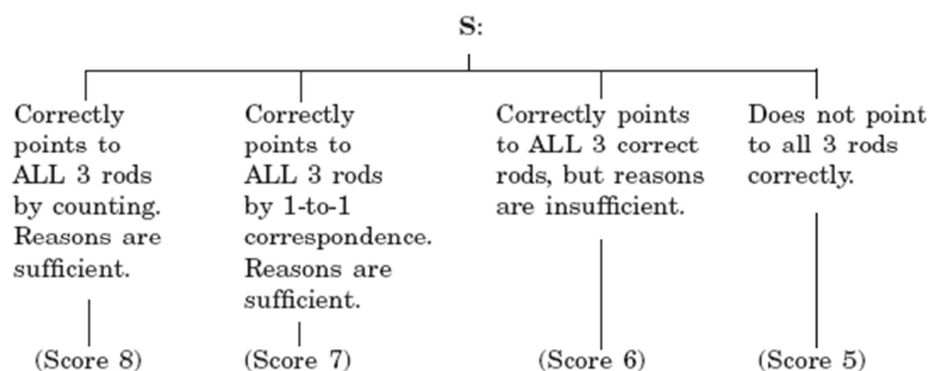
“I am going to point to a circle and I want you to point to the stick that goes with that circle.” (Throughout this part of the task the sticks must remain in the ordered set, they may not be moved. The **S** may move or rearrange the circles if he/she wants to do so.)

The **I** next points to three of the circles, one at a time, in the order given below.

1. The **I** points to the SMALLEST YELLOW CIRCLE (I).
2. The **I** points to the LARGEST BLUE CIRCLE (C).
3. The **I** points to the MIDDLE-SIZED RED CIRCLE (E).

As the **I** points to each circle the **I** should say, “Point to the stick that goes with this lollipop.”

Since there is a chance for guessing the **I** must *always* ask, “Why did you pick that one?” after each selection.




---

### Response Categories

Score	Response
0	The S does not pick shortest and longest rods. Task terminated.
1	The S selects the shortest and longest rods, but does not order Set A correctly. Task terminated.
2	The S orders Set A, but cannot insert ANY additional rods. Task terminated.
3	The S orders Set A and inserts at least ONE additional rod from Set B. Does not insert ALL 3 rods from Set B correctly. Task terminated.
4	The S orders Set A, inserts ALL 3 rods from Set B correctly, but does NOT match correct rods and circles, i.e., makes two or more errors in the matching. Task terminated.
5	The S orders Set A, inserts all 3 rods correctly, and matches correct rods with circles, but does not point to all three correct rods.
6	The S orders Set A, inserts all 3 rods correctly, and matches correct rods with circles, but one or more reasons for the selections are not sufficient.
7	The S orders Set A, inserts correctly, matches correct rods and circles and points to correct rods by using 1-to-1 correspondence. The reasons for the selections are sufficient.
8	The S orders Set A, inserts correctly, matches correct rods and circles and points to correct rods by counting. The reasons for the selections are sufficient.



**RESPONSE SHEET**  
for  
**Addition of Asymmetrical Relations (LG<sub>5</sub>)**  
**(Ordering Rods & Circles)**

Interviewer: \_\_\_\_\_ Date: \_\_\_\_\_.

Subject I.D.: \_\_\_\_\_ Age: \_\_\_\_\_ yrs. \_\_\_\_\_ mons.

---

**PART A — Ordering Set A**

S selects shortest and longest sticks: Yes \_\_\_\_ No \_\_\_\_ (Terminate)

S orders Set A correctly: Yes \_\_\_\_ No \_\_\_\_ (Terminate)

---

**PART B — Inserting into the existing set**

S inserts 3 rods from Set B correctly: Yes \_\_\_\_ No \_\_\_\_ (Terminate)

---

**PART C — Ordinal correspondence between two sets**

S makes more than one error in the  
matching (more than 2 rods out of place): Yes \_\_\_\_ (Terminate)

S matches all circles & rods correctly, or  
makes only one error: Yes \_\_\_\_

---

**PART D — Maintaining correspondence between two ordered sets**

S does not point to all three correct rods: \_\_\_\_

S points to all three correct rods, but  
reasons are insufficient: \_\_\_\_

S points to all three correct rods, and  
reasons are sufficient: \_\_\_\_

Method used to select correct rods: 1-to-1 correspondence \_\_\_\_  
counting \_\_\_\_

---

Response Category: \_\_\_\_\_

APPENDIX E:  
EQUIPMENT, PROCEDURE, SCORING CRITERIA, AND SCORING SHEETS  
FOR TASK 4--CONSERVATION OF NUMBER

**INTERVIEW PROTOCOL**  
**for**  
**Conservation of Number**

**Equipment**

18 checkers, small tiles, pieces of paper, flat blocks, etc. (Note: These should be a solid color. 10 should be of one color, 8 of a different color.)

**Preparation**

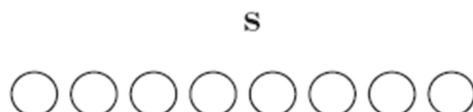
The blocks should be separated into 2 groups by color.

Note: Do not use the word “counter” in this task. It is a cue and may lead the child to the insufficient response of counting.

**Procedure**

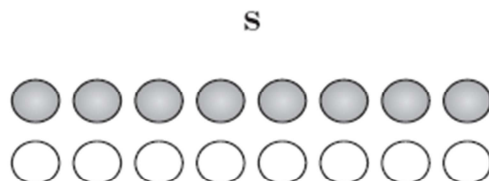
**PART A — Establishing equivalence**

- I:** “Here I have some blocks and I am going to put them like this.” (The **I** places 8 white blocks in a row as shown in Figure 1. There should be a space between each block.)



**I**  
**Figure 1**

- I:** “Now I’m going to give you these blocks.” (The **I** gives the **S** the 10 green blocks.) “I would like for you to make a row of these blocks beside the row of white blocks so that there is one green block for each white block.” (The **I** should indicate by pointing that the **S**’s row should be beside the white blocks, nearest the **S**.)



**I**  
**Figure 2**

S: (Responds)

I: "Is there one white block for each green block, or not?" (The *order* in which the colors are stated in this question is important. The **I** should *not* ask, "Is there a green block for each white block?" If the **S** has put all 10 green blocks beside the row of 8 white then there certainly *is* a green block for each white block. There are two extra green blocks, but that is not the question being asked.)

S: (Responds)

(If the **S** has not made a correct 1-to-1 correspondence as shown in Figure 2, and if when asked the question above the **S** says, "No," the **I** may then say, "Can you make it so there is a white one for each green one?" If the **S** can correct the error the task continues, if not the task should be terminated. If the task continues the **I** should remove the two extra green blocks from sight.)

#### PART B — Group versus Original Row

I: "Now watch what I do. I'm going to move my row like this. (The **I** moves the white row into a group as shown in Figure 3.)

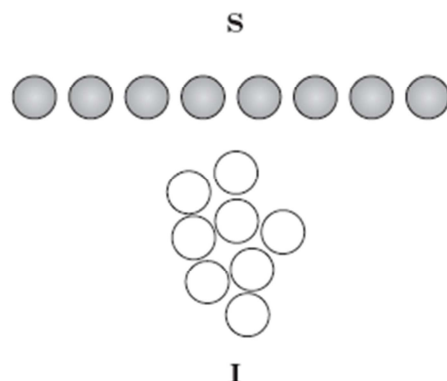


Figure 3

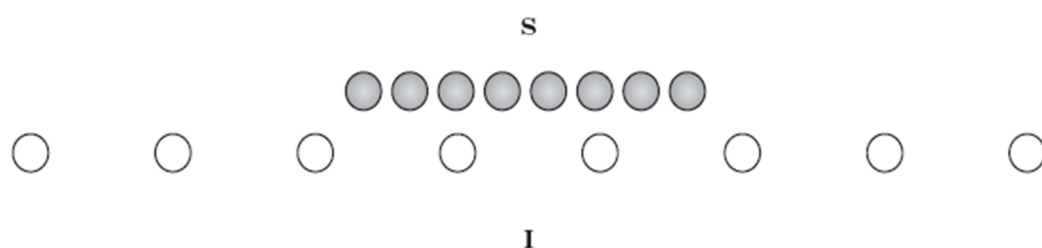
I: "Tell me. Are there more green blocks, more white blocks, or are there the same number of green and white blocks?"

S: (Responds)

I: "Why do you think so?" (The **I** may pursue an insufficient reason *if the S answered "Same."*)

#### PART C — Short Row versus Long Row

I: "Now watch what I do. I'm going to move the white ones like this." (The **I** moves the white blocks into a row at least *two or three times as long* as the green row as shown in Figure 4.)



**Figure 4**

- I: "Look at all the blocks. Are there more green blocks, more white blocks, or are there the same number of green and white blocks?"
- S: (Responds)
- I: "Why do you think so?" (The I may pursue an insufficient reason *if the S answered "Same."*)

---

### Insufficient Reasons

The examples given below are typical of the types of responses children give to this task. These "reasons" are not incorrect; they are insufficient. If a child can only give responses such as those given below, then that child cannot be scored as passing that task part.

<u>Type</u>	<u>Example</u>
Counting	"There's 8 white ones and 8 green ones."
Description	"This row's short and this row's long."
Action	"You just moved those into a pile."
Empirical	
Reversibility	"If you moved them back it would be the same number."
Identity	"They're still the same blocks."
Repeat of Choice	"I think there's the same number."

### Response Categories

<u>Category</u>	<u>Description</u>
0	The S did not make a correct 1-to-1 correspondence. Task terminated.
1	The S made a correct 1-to-1 correspondence in Part A, but could not do Parts B or C.
2	The S did Part A and stated correct <i>choices</i> for Parts B and C, but did not give sufficient reasons for either Part B or Part C.
3	The S did Part A and Part B (correct choice <i>and</i> reason), but could not do Part C.
4	The S did Part A and Part C (correct choice <i>and</i> reason), but could not do Part B.
5	The S did all Parts of the task correctly. Note that to pass Parts B and C the S must provide a correct choice <i>plus a logical reason</i> .

**RESPONSE SHEET**  
for  
**Conservation of Number ( $N_1$ )**

Interviewer: \_\_\_\_\_ Date: \_\_\_\_\_.

Subject I.D.: \_\_\_\_\_ Age: \_\_\_\_\_ yrs. \_\_\_\_\_ mons.

---

**PART A — Establishing Equivalence**

S made the 1-to-1 correspondence:      Yes \_\_\_\_ No \_\_\_\_ (Terminate)

---

**PART B — Group versus Row**

S's Response: More in Row \_\_\_\_ More in Group \_\_\_\_  
Same \_\_\_\_

Reason:

---

**PART C — Short Row versus Long Row**

S's Response: More in Long Row \_\_\_\_ More in Short Row \_\_\_\_  
Same \_\_\_\_

Reason:

---

Response Category: \_\_\_\_\_

APPENDIX F:  
HUMAN SUBJECTS AUTHORIZATIONS





Office of the Dean  
The Graduate School

## The Union Institute

440 East McMillan Street  
Cincinnati, Ohio 45206-1947  
513/861-6400 ♦ 800/486-3116  
TDD 800/486-9968 ♦ FAX 513/861-0779

October 18, 1996

Keith Bookwalter  
Calle 66 #50-97  
Barranquilla, Colombia

Dear Keith,

Thank you for your letter dated September 21, 1996.

We sincerely hope that your program continues to progress with productivity and stimulation.

I understand the situation regarding the "informed consent" for your research subjects. After discussing your situation with Assistant Dean, Gail Brophy, we hereby affirm your proposal to waive the need for written permission and informed consent from the parents of each of the children. In your circumstance, it will be permissible to utilize the permission from the principal and the administrative council at the schools in lieu of obtaining the individual permissions from the parents of each of the children. The waiver of the need for individual parental consent is based upon the following assumptions:

1. The interview process and procedures which you propose involve activities which are similar to those activities generally engaged in the process of the school day.
2. The interview questions which you will utilize with the children are non-invasive, i.e. they do not involve exploration into personal or family issues which would not ordinarily be engaged in the process of the school day.

We wish you the best with your research project.

Sincerely,

Lawrence J. Ryan, PH. D.  
Dean

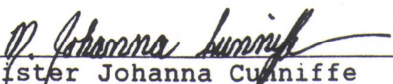
LJR/cam

April 1, 1996

To: Keith Bookwalter  
From: Sister Johanna Cunniffe  
Re: Parental permission for the clinical interviews which  
form part of your Union Institute research project

Because clinical interviews which assess students' cognitive competence are an on-going function of our psychology department and these assessments do not require prior parental consent, the School is giving you permission to proceed with your Union Institute research project as part of our regular psychometric program and without the necessity of obtaining prior parental approval.

It is understood that all of your research findings will be confidential and that no names will be used in any resulting report or publication.

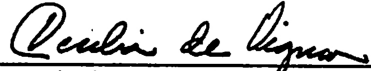
  
Sister Johanna Cunniffe  
Director  
Marymount School  
Barranquilla, Colombia

April 15, 1996

To: Keith Bookwalter  
From: Cecilia de Vigna  
Re: Parental permission for the clinical interviews which form  
part of your Union Institute research project

Because the assessment of students' cognitive ability is an on-going aspect of our educational program in the Fe y Alegría Schools and these assessments do not require prior parental consent, I am giving you permission to proceed with your Union Institute research project as part of our regular assessment program and without the necessity of obtaining prior parental approval.

It is understood that all of your research findings will be confidential and that no names will be used in any resulting report or publication.



Cecilia de Vigna  
Regional Director  
Fe y Alegría

APPENDIX G:  
DATA SUMMARY

## APPENDIX G: DATA SUMMARY

DATA SUMMARY FOR AGE RANGE 5.6 - 6.5 YEARS							
Subject Information					Scores on Tasks		
					Task 1	Task	Task 3(s/8)
Subject No.	School*	Age	Gender	Collections	Class Inc.	Ordering	Cons. No.
1	MM	5.8	F	GN	3	1	2
2	MM	5.11	F	NG2	3	1	1
3	MM	5.10	F	NG3	3	5	5
4	MM	6.4	F	NG2	5	0	2
5	MM	5.8	F	G1,6a	5	1	2
6	MM	6.1	F	NG2	1	1	1
7	MM	6.1	F	NG3	5	4	5
8	MM	6.0	F	G6b	5	1	2
9	MM	5.9	F	G1.3	5	1	5
10	MM	5.9	F	NG2	5	3	1
11	MM	5.11	F	NG3	5	8	1
12	MM	5.11	F	G1	5	1	1
13	MM	5.8	F	NG2	1	1	2
14	MM	5.10	F	NG3	5	1	2
15	MM	5.8	F	NG2	5	1	2
18	MM	6.3	F	NG2	2	2	2
17	MM	6.5	F	NG2	5	3	2
18	MM	5.8	F	NG2	5	3	1
19	MM	6.3	F	NG3	8	1	0
20	MM	5.7	M	NG2	3	1	2
21	MM	6.4	M	G6a,3	5	1	1
22	MM	5.7	M	NG3	5	0	0
23	MM	6.5	M	NG2	8	5	3
24	MM	6.1	M	GN	5	5	2
25	MM	5.8	M	GN	5	3	1
26	MM	6.5	M	NG3	5	1	3
27	MM	6.1	M	G3.6a.5	5	3	1
28	MM	6.0	M	NG2	5	3	5
29	MM	5.8	M	NG2	5	1	1
30	MM	6.0	M	NG2	5	1	1
31	MM	6.2	M	NG2	5	1	4
32	MM	5.8	M	G6b	1	1	1
33	MM	6.2	M	NG2	1	1	0
34	MM	5.10	M	G1	5	1	0
35	MM	5.7	M	NG2	1	5	2
36	MM	5.1	M	NG2	5	1	2
37	MM	6.3	M	NG2	5	5	5
38	MM	5.1	M	NG2	2	2	0
39	MM	6.0	M	G1.4	5	1	2
40	MM	5.10	M	NG2	5	1	0

\*FA = Fe y Alegría School

\*MM = Marymount School

## APPENDIX G: DATA SUMMARY

DATA SUMMARY FOR AGE RANGE 6.6-7.5YEARS							
Subject Information					Scores on Tasks		
					Task 1	Task	Task 3(s/8)
Subject No.	School	Age	Gender	Collections	Class Inc.	Ordering	Cons. No.
41	FA	6.10	F	G3,4	5	5	5
42	FA	7.2	F	NG2	5	4	5
43	FA	7.2	F	G1,4	5	8	1
44	FA	7.3	F	G1,3	5	1	2
45	FA	7.3	F	NG3	2	1	2
46	FA	7.1	F	NG3	2	1	0
47	FA	6.9	F	G1,3	2	0	1
48	FA	7.4	F	NG3	5	1	2
49	FA	7.5	F	G1,4	5	1	1
50	FA	7.0	F	NG2	5	1	2
51	MM	6.9	F	NG2	5	0	2
52	MM	7.1	F	NG2	8	5	2
53	MM	6.7	F	NG2	2	1	1
54	MM	6.7	F	G1	5	4	2
55	MM	7.0	F	G4,2	5	8	2
56	MM	6.6	F	NG2	5	5	3
57	MM	6.10	F	NG2	5	5	3
58	MM	6.9	F	NG3	5	5	1
59	MM	7.2	F	NG2	5	5	5
60	MM	6.11	F	NG3	5	4	2
61	MM	6.9	F	GN	5	6	2
62	MM	6.8	F	NG2	5	5	2
63	MM	6.6	F	G1,3	1	3	1
64	FA	6.6	M	NG2	5	1	1
65	FA	7.2	M	NG2	3	1	1
66	FA	7.1	M	NG2	5	4	5
67	FA	6.9	M	NG2	5	3	4
68	FA	7.0	M	G4	0	1	2
69	MM	7.2	M	NG2	5	8	5
70	MM	6.10	M	G1,4	5	6	2
71	MM	7.3	M	NG2	5	5	4
72	MM	6.6	M	GN	5	5	4
73	MM	6.8	M	NG3	5	5	5
74	MM	6.7	M	G6b,3	5	5	3
75	MM	6.8	M	NG2	8	5	2
76	MM	7.2	M	G6b,4	5	5	2
77	MM	7.2	M	NG3	5	1	1
78	MM	6.11	M	NG2	8	8	2
79	MM	6.7	M	NG2	8	5	2
80	MM	6.11	M	NG2	5	3	2

APPENDIX G: DATA SUMMARY

DATA SUMMARY FORAGERANGE 7.6-8.6YEARS							
Subject Information					Scores on Tasks		
					Task 1	Task	Task 3(s/8)
Subject No.	School	Age	Gender	Collections	Class Inc.	Ordering	Task 4(s/5)
81	FA	7.8	F	NG2	5	1	2
82	FA	8.3	F	NG2	5	5	1
83	FA	8.4	F	NG2	5	1	1
84	FA	8.5	F	GN	5	4	1
85	FA	8.6	F	NG2	5	7	1
86	FA	8.2	F	NG2	1	1	0
87	FA	8.4	F	NG2	2	1	2
88	FA	7.6	F	G1,3	5	4	1
89	FA	8.1	F	G1,3	5	4	1
90	FA	7.9	F	NG2	2	4	1
91	FA	8.1	F	GN	5	4	2
92	FA	8.0	F	NG2	5	4	1
93	MM	7.8	F	NG3	5	5	1
94	MM	7.10	F	NG3	5	7	1
95	MM	8.2	F	NG2	5	5	1
96	MM	8.1	F	NG2	8	5	2
97	MM	8.0	F	NG2	2	1	1
98	FA	7.7	M	G6b	5	1	2
99	FA	7.7	M	NG2	1	0	0
100	FA	7.7	M	NG2	1	1	2
101	FA	7.8	M	NG2	1	0	2
102	FA	8.1	M	G3	5	3	2
103	FA	8.5	M	NG2	5	1	1
104	FA	8.0	M	NG2	5	3	2
105	FA	8.2	M	G1,4	5	0	0
106	FA	8.5	M	NG2	5	1	2
107	FA	8.0	M	NG2	2	1	2
108	FA	8.0	M	G1,2	1	1	1
109	FA	7.7	M	G1,4	1	1	0
110	FA	7.9	M	NG2	5	1	2
111	FA	8.3	M	NG2	5	4	2
112	MM	7.7	M	NG3	5	5	2
113	MM	8.4	M	NG2	5	5	5
114	MM	8.2	M	NG2	8	4	2
115	MM	8.3	M	NG2	5	8	3
116	MM	7.6	M	G1	5	7	2
117	MM	7.10	M	NG2	5	4	1
118	MM	78	M	NG2	8	5	2
119	MM	7.9	M	NG2	5	5	5
120	MM	7.8	M	G1,4	3	5	4

APPENDIX H:  
JEAN PIAGET -- A BIOGRAPHICAL SKETCH



## JEAN PIAGET: A BIOGRAPHICAL SKETCH

The following is a cursory timeline of some of the pertinent events in the life of Jean Piaget (Crain, 1985, pp. 88-90; Flavell, 1963, pp. 1-9; Ginsburg and Oppen, 1988, pp. 2-12) followed by a few glimpses of Piaget, the man, as seen through the eyes of David Elkind.

1896 - Born in Neuchatel, Switzerland. Father was medieval historian at local university. Mother is described as dynamic, intelligent, religious, highly emotional and conflictive causing Jean to seek refuge in solitary research.

1906 - Published first article at age of 10 on albino sparrow he had seen in park.

1911-13 - Began study of mollusks and published a series of articles. (These studies of how mollusks develop differently under varying conditions would later contribute to his conception of the expression of the human phenotype as being the result of the interaction between the genotype and the environment.)

1911 - Began intellectual search for a scientific foundation to undergird his religious and philosophical convictions.

1917 - Earned his doctorate in natural sciences at age of 21.

1919 - Began scientific research in child psychology using his findings to create "genetic epistemology," an integration of epistemology (as a traditional branch of philosophy concerned with the origin of knowledge) with developmental psychology.

1920 - Began studying children while working at the Binet Laboratory in Paris. Became disenchanted with intelligence tests requiring only right/ wrong answers. Became fascinated with children's illogical "wrong" answers and began the development of his clinical interview technique.

1921 - Accepted offer to become director of research at the Jean-Jacques Rousseau Institute in Geneva. (Later became co-director.)

1923 - Published book on language and thought in the child.

1924 - Published book on judgement and reasoning in the child.

1925 - First child was born. He and wife, Valentine Chatenay, began important series of observations of their three infants. Continued study of children's dreams, morality and other themes of interest to the child.

1929-39 - Appointed professor of history of scientific thought at Geneva University.

- Taught experimental psychology at Lausanne University.
- Accepted chairmanship of International Bureau of Education (later to be affiliated with UNESCO).
- Followed Albert Einstein's advice to investigate the child's understanding of time, velocity, and movement.
- Turned to the study of the development of logical, mathematical, and scientific concepts in the child.

1950 - Turned again to philosophical issues in epistemology. 1960 - His work experienced a revival of interest.

1980 - Jean Piaget passed away at the age of 84. His work is criticized by some and built upon by others. His legacy is presently being felt most greatly in the constructivist movement (which has several splinter groups) and the increasing use of the clinical interview technique for research, assessment, and mediated instruction.

Piaget's investigations of children's thought processes covered an incredibly productive 60 years which resulted in the publication of 60 books and hundreds of articles. But behind this image of an intellectual giant was a remarkably sensitive, humble, and endearing human being. Amongst the several biographers that this author

reviewed, the description of David Elkind (1974) stands out as unique.

Elkind had the privilege of spending a year (1964-1965), at Piaget's invitation, at the Institute of Educational Science in Geneva. Elkind (1974) describes Piaget's physical appearance in the following way:

[Piaget] is an arresting figure. He is tall and somewhat portly, and his stooped walk, bulky suits, and crown of long white hair give him the appearance of a thrice-magnified Einstein. His personal trademarks are his meerschaum pipes (now burned deep amber), his navy blue beret, and his bicycle. (p. 12)

Elkind also gives us a glimpse of Piaget's daily routines:

He arises early each morning sometimes as early as 4 a.m., and writes four or more publishable pages. Later in the morning he may teach classes and attend meetings. His afternoons include long walks during which he thinks about the problems he is currently confronting. He says, "I always like to think on a problem before reading about it." In the evenings, he reads and retires early. Even on his international trips, Piaget keeps to this schedule. (p. 13)

Another routine which occurred on a yearly basis was Piaget's summer retreats. When Elkind (1974) was writing his description, Piaget was seventy-three and had been spending the last fifty summers at an abandoned farm house in the Alps. Apparently Piaget felt the need to balance his intense work with human beings with periods of solitude. He went alone to his retreat and only an inner circle of trusted colleagues knew of his whereabouts. He took with him the research results of his assistants and spent the summer meditating, taking long walks, and writing. Each year he would descend from the mountains, like a grand Moses, with books and articles he

had written during his "vacation."

When Elkind met Piaget in person he found a man of abundant "Old-World charm and graciousness" who emanated "an aura of intellectual presence not unlike the aura of personality presence conveyed by a great actor" which, Elkind believed, was felt by everyone. He described Piaget's ability, when listening to Elkind's seemingly superficial remarks, to detect "a significance and depth" of which he had not been consciously aware (Elkind, 1974, p. 13).

This characteristic of genius described by Elkind (1974, p. 13) as the ability "to search for relevance in the apparently commonplace and frivolous" was the hallmark of many of Piaget's most remarkable discoveries. Where we might perceive an ordinary, childish behavior or "slip of the tongue" Piaget saw as an entry point into the mind of the child. A few examples will illustrate this point.

As mentioned earlier, while working at Alfred Benet's laboratory school in Paris, Piaget became fascinated with children's wrong answers. Apparently it was a task similar to the class inclusion task which provoked his interest (Phillips, 1995). When presented, for instance, with 9 petunias and 4 roses the child is asked whether there are more flowers or more petunias. Younger children will often say that there are more petunias. For many adults this might seem to be just a curiously wrong, childish answer. But for Piaget it became a window revealing that the world view of children is very different from that of adults.

In order to further explore the conceptions of children, Piaget moved away from paper and pencil tests and began using a semi-clinical, open-ended interview procedure which he had learned during a brief internship at a psychiatric clinic in Zurich. His investigations revealed, for example, that many youngsters believe that the sun and the moon follow them when they are out walking, "that anything which moves is alive, that the names of objects reside in the objects themselves and that dreams come in through the window at night" (Elkind, 1974, pp. 14-15). Piaget conducted a

large number of these interviews which led to his publication of an article entitled "Children's Philosophies."

Piaget was not only a close observer of statements, he also gained insights from watching children play. It was at the Maison des Petits school in Paris where he noticed a "peculiar lack of social orientation" of the children who would often talk *at* rather than *to* one another, chattering about unrelated topics and using invented words which could have only had personal meaning to the child. It was from these simple, initial observations that he began to formulate his theories about egocentrism over a period of several decades (Elkind, 1974, p. 15-16).

Piaget was also a close observer of his own children--Jacqueline, Lucienne, and Laurent. He noticed, for instance, that as infants of 4 or 5 months, when playing with a toy which rolls out of sight, they ceased to look for it, even when it was within reach. They behaved as though the toy had ceased to exist, not just gone out of sight. This and many other such subtle observations led Piaget to develop the concept that he termed "conservation of the object" and to the publication of his books *The Origins of Intelligence in Children*; *Play, Dreams and Imitation*; and *The Construction of Reality in the Child*.

One day Piaget took his son Laurent for a drive. As they were driving along Laurent asked the name of the mountain that was off to one side. For most parents this may have seemed like a normal question for an inquisitive child. But Piaget was thoroughly intrigued, for he knew that this mountain called Saleve, which dominates the city of Geneva, was familiar to Laurent because he could see it from his yard where he played every day. Piaget realized that the change of perspective had led Laurent to believe he was looking at an entirely different mountain. This incident, along with other factors, led Piaget to investigate children's concept of space (Elkind, 1974).

There are many other similar stories which reveal Piaget's sensitive observations of children's behavior and comments, observations which became for Piaget the grist

for sixty years of theorization and experimentation. His work has received both acclaim and criticism. But it was the latter case which revealed, in this author's opinion, the humbleness of his character, for he was, as described by Elkind, "remarkably open" to the serious responses of his critics. He even went so far as to invite them to spend a year in Geneva in order to discuss differences and study them in depth. He had no desire to develop a following of "dogmatic believers" and once remarked, "To the extent that there are Piagetians, to that extent have I failed" (as cited in Elkind, 1974, p. 27).

This sketch of Piaget concludes with a description of an incident related by Elkind (1974) which reveals the special, endearing relation that Piaget had with children. In 1967 Piaget had been invited to deliver the Heinz Werner Memorial Lectures at Clark University in Worcester, Massachusetts. Elkind was invited one evening, before the lectures had begun, to dine with Piaget and others. He noted a certain apprehensiveness in Piaget's demeanor, a certain lack of his usual liveliness and charm, which he attributed to two factors: Piaget's awareness of the historical significance of Sigmund Freud's having also lectured at Clark in 1909 and that he would be speaking through a translator to a huge American audience. Then an interesting incident occurred:

About half way through the meal there was a small disturbance. The room in which the dinner was held was at a garden level and two boys of seven or eight suddenly appeared at the windows and began tapping at them. The inclination of most of us, I think, was to shoo them away. Before we had a chance to do that, however, Piaget had turned to face the children. He smiled up at them, hunched his shoulders and gave them a slight wave of his hand. They hunched their shoulders and smiled in return, gave a slight wave, and disappeared. After a moment, Piaget turned back to the table and began telling stories and entering

into animated conversation. (p. 12)

Elkind felt that even though he was sure that Piaget's lectures would have been a great success regardless of this incident, nevertheless, "the encounter with the boys did much to restore his vigor and good humor" (p. 12). This story did much to bring the spirit of Piaget close to the heart of this author who also finds working with children to be a most invigorating and heart-warming experience; one to which it is well worth dedicating a lifetime.

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